The Palindrome Triptych
For Chamber Trio and Percussion (1 player)

Remembrance Day Sunday Concert
The Great Hall
Liverpool Hope University Creative Campus

The Pixels Ensemble
Monday 13th November 2017

Initial Premise

… " To maximise variety, therefore, I'd be grateful if you'd consider contributing a piece for violin, cello, clarinet and percussion. Obviously you can do whatever you like, but it would be useful thematically if you'd consider somehow responding to an aspect of the Messiaen as a starting point. Length-wise I think we're looking at about 8-10 minutes to create a balanced first half of: your piece, the Lentz and one further piece. The second half will be quite a bit longer, as the Messiaen lasts around 65 minutes …

Recital Programme

Ian Percy – Palindrome Triptych for clarinet, violin, violoncello and percussion (1 player)
Toru Takemitsu – Rain Tree Sketch II (in memoriam Olivier Messiaen), for solo piano
Louis Andriessen - Woodpecker, for solo percussion (marimba, wood and temple blocks)
Georges Lentz – Ngurra for clarinet, violin, violoncello, piano and percussion (1 player)
Olivier Messiaen – *Quatuor pour la fin du temps* (Quartet for the End of Time) for clarinet, violin, violoncello and piano (1940-41)

Primary Context

Messiaen – Time – WWI Centenary (1914-18) – Remembrance Sunday.

Fundamental Considerations

Messiaenic references

i. Pitch Palindromes (vertical and linear, harmonic and melodic).
ii. Modes of Limited Transposition (palindromic and equidistant modes).
iii. Time and Eternity – Meter, Pulse, Space and Distance (tempo modulation).
iv. Rhythm Palindromes – Non-retrogradable Rhythms (vertical and linear).
v. Unmetered and Additive Rhythms (augmented and diminishing phrases).
vi. Birdsong (bowed glissando, trills, string harmonics, echoes and imitations).
vii. Eulogy (extended thematic writing and melodic contours).
viii. Vagrant Chords, Ambiguous and Extended Harmony, Resonance and Decay.

Pitch Organisation

(Palindromes – From the Micro to the Macro)

1. Unison (Octave) – The details of the individual note:

   i. The unison is a static palindrome frozen in time (vertical time).
   ii. An isolated pitch is therefore (theoretically) the smallest unit through which to present a symmetrical, self-contained, vertical and linear, pitch and rhythm palindrome.
   iii. In equal temperament, there are 12 individual pitch-classes numbered 0-11.
   iv. When employed as a drone, the unison can be given linear momentum through the use of dynamic, timbre, duration, register, density and intensity.
   v. When employed as an ostinato, the added manipulation of rhythm and articulation offers further dimensions for establishing linear motion (goal) and introduces the possibilities for musical expression on a single note.
   vi. Messiaen employs the unison (octave) to compose homophonic monophony (tutti unison and octave) within his *Quartet for the End of Time*.
   vii. Therefore, the unison can be used across the ensemble as a vertical or horizontal palindrome, with or without linear motion.
   viii. The unison ostinato is useful for exploring palindromic (non-retrogradable) rhythms and potential sketches for augmenting and diminishing phrasing.

2. Intervals, Interval-types and Interval-classes – Notes in Pairs (linear or vertical):

   i. As isolated units, all intervals (paired-pitches, interval-types) are self-contained <u>static equidistant palindromes</u>.
   ii. There are twelve interval-types (intervals): unison/octave, minor 2^{nd} (semi-tone), Major 2^{nd} (whole-tone), minor 3^{rd}, Major 3^{rd}, Perfect 4^{th}, tritone (diminished 5^{th}), Perfect 5^{th},minor 6^{th}, Major 6^{th}, minor 7^{th} and Major 7^{th}.
   iii. The twelve interval-types divide into six interval-classes.
   iv. All interval-classes invert to the same interval-class (palindromes).
   v. Interval-classes function like harmonic inversions in that the inversion completes the span of an octave, therefore the whole-tone inverts to a minor 7^{th} and a minor third inverts to a Major 6^{th} etc.
   vi. Interval classes 1 – 5 are all <u>asymmetrical palindromes</u> (mirror opposites).
   vii. Interval class 6 is a <u>symmetrical palindrome</u> (mirror symmetry).
   viii. Interval classes 1, 2, 3, 4 and 6 can all be used in multiple to complete an <u>octave equidistant scale</u>; a self-contained and <u>symmetrical cyclic palindrome</u>:
      a. Interval Class 1 (semi-tone/Major 7^{th}): 12 semi-tones = 1 octave.
      b. Interval Class 2 (whole-tone/minor 7^{th}): 6 whole-tones = 1 octave.
      c. Interval Class 3 (minor 3^{rd}/Major 6^{th}): 4 minor thirds = 1 octave.
      d. Interval Class 4 (Major 3^{rd}/minor 6^{th}): 3 Major thirds = 1 octave.
      e. Interval Class 6 (tritone/diminished 5^{th}): 2 tritones = 1 octave.

3. Interval Class Five: 2-5 (05) <000010> Perfect Fourth: Cycle of Fourths

    i. Interval class five (Perfect $4^{th}/5^{th}$) is unique in the fact that multiple repetitions do NOT equate to the span of a single octave.
    ii. The cycle of fourths takes a span of five octaves to return to the unison (octave) and complete the equidistant palindrome (cycle).
    iii. The cycle of fourths passes through all twelve pitch-classes and produces all twelve individual interval-types (without replication) in each rotation.
    iv. The cycle of fourths is therefore an equidistant all-interval consonant twelve-tone scale (chromatic consonance).
    v. The cycle of fourths can be used as a linear (melodic) or vertical (harmonic) self-contained equidistant and infinite pitch palindrome.
    vi. Interval-classes 1 and 5 are of particular note, because they both complete the total chromatic (twelve-tone scale) in one cyclic rotation: Class 1 results in an audible cycle of total dissonance and class 5 results in an audible cycle of total consonance.

4. Interval Class Six: 2-6* [6] (06) <000001>[1] Tritone: Mode VII Complement

    i. Interval class 6 (tritone) is unique in the fact that it inverts to the same interval-class AND interval-type.
    ii. The tritone (augmented 4^{th}/diminished 5^{th}) divides the octave in two and is therefore the smallest possible unit through which to present the octave as a perfect symmetrical pitch palindrome around a literal pivot-note (axis).
    iii. Although microtonality is not considered within the processes for this piece, it is worth note that the tritone is theoretically an equidistant division of the octave often heard as a consonant interval in music using 'just-intonation'.
    iv. The name of the interval suggests that it is already a self-contained palindrome: tritone = three tones (also two minor 3^{rds}).
    v. Interval class 6 is the complement to Messiaen's Mode VII (Decachord 10-6)
    vi. Interval class 6 MUST be used as the pivot interval in order to produce a symmetrically inverted all-interval twelve-note (SI AITN) chord or row.
    vii. The tritone is often referred to as the Lydian Augmented Fourth.

5. Trichord 3-12* [4] (048) <000300> Augmented Triad: Mode III Complement

    i. The augmented triad is the only triad to complete the octave and produce an equidistant pitch palindrome. It is often referred to as a vagrant chord.
    ii. The augmented triad only contains interval-class 4: 3 Major 3^{rds} = 1 octave.
    iii. Trichord 3-12 (048) is complement to Messiaen Mode III (Nonachord 9-12).
    iv. Trichord 3-12 is the only trichord to be highlighted within these processes.
    v. Trichord 3-12 subsumed within 6-14 and so belongs to secondary resources.

[1] * Indicates that the set is a mirror. The integers in square parenthesis indicate limited transposition.

6. Tetrachord 4-9* [6] (0167) <200022> Double Tritone Tetramirror: Messiaen Mode IV Complement

    i. Tetrachord 4-9 is the complement to Messiaen Mode IV (Octachord 8-9).
    ii. Tetrachord 4-9 (0167) is a self-contained palindrome (mirror).

7. All-Interval Tetrachords (AIT): 4-z15 (0146/0256) & 4-z29 (0137/0467) <111111>

    i. The two All-Interval Tetrachords (AIT) get their name due to the fact they contain all twelve interval-types without replication:
        a. 0146/0256 (4-z15/4-z15B) <111111> All-Interval Tetrachord 1
        b. 0137/0467 (4-z29/4-z29B) <111111> All-Interval Tetrachord 2
    ii. This document has defined that the interval is a micro palindrome and that all interval-classes can produce equidistant pitch palindromes. The AIT are therefore the most succinct way to reference the theoretical concept of intervallic pitch palindromes within a single pitch-class set.
    iii. The all-interval tetrachords (AIT) are self-contained palindromes.
    iv. *The Pictures on Your Wall* for Trombone and Percussion (2003-09) uses a symmetrical and asymmetrical (primary and secondary) cycle of chord inversions as the primary pitch resource. This is a form of harmonic palindrome that should be consulted for this research. The second movement also (briefly) explores the concept of all-interval composition.
    v. *Natural Causality* (2008-09) and *An Instant Conception* (2009) for chamber ensemble and percussion explore the articulation, expression and timbre of the individual pitch and primary characteristics of interval-types. The interval is the fundamental resource for pitch organisation, form and proportion.
    vi. *Ensuen* for small ensemble (2009-10) uses the AIT as the fundamental resource for pitch organisation, proportion and form (consult research).

8. Tetrachord 4-25* [6] (0268) <020202>: French Sixth Chord: Mode VI Complement

    i. Tetrachord 4-25 is the complement to Messiaen Mode VI (Octachord 8-9).
    ii. Tetrachord 4-25 (0268) is a self-contained palindrome (mirror).

9. Tetrachord 4-28* [3] (0369) <004002> Diminished Seventh Chord – Vagrant Chord Messiaen Mode II Complement: Not subsumed in any set smaller than octachord

    i. The Diminished 7^{th} Chord is the only tetrachord to complete the octave and produce an equidistant pitch palindrome: 4 minor 3^{rds} = 1 octave.
    ii. The Diminished 7^{th} Chord only contains two interval-classes (class 3 and 6) and theoretically divides the octave into equidistant halves and quarters.
    iii. Tetrachord 4-28 (0369): Complement to Messiaen Mode II (Octachord 8-28).
    iv. 4-28 can be a neutral transition or part of the primary & modal/tonal family

10. Pentachord 5-35* (02479) <032140> Major Pentatonic Scale – Black Key Pentatonic/Quartal Pentamirror: Dorian mode complement & subsumed within 6-32

    i. Pentachord 5-35 is complement to Dorian minor mode (Heptachord 7-35).
    ii. Pentachord 5-35 has obvious tonal/modal foundations and is a literal spelling of the Major Pentatonic Scale: 2-2-3-2-3: C-D-E-G-A-C (02479).
    iii. Pentachord 5-35 (02479) is a self-contained palindrome (mirror).
    iv. Pentachord 5-35 (02479) can be used as a vehicle for a 'rotating' modulation and an alternative to the classical concept of functioning harmony.
    v. Each transposition of the pentachord produces a different modulation:
        a. C Major Pentatonic modulates through inversion to A^{b} Major (minor 6^{th}).
        b. D Major Pentatonic modulates through inversion to $F^{\#}$ Major (Major 3^{rd}).
        c. E Major Pentatonic modulates (inverts) to E Major (unison modulation).
        d. G Major Pentatonic modulates through inversion to D^{b} Major (tritone).
        e. A Major Pentatonic inverts through modulation to B Major (Major 2^{nd}).
    vi. This unusual characteristic of rotating modulation through inversion of the Major Pentatonic Scale should be explored in more detail within future composition/research. See Modal Inversions Document.
    vii. Pentachord 5-35 (02479) – Major Pentatonic Scale is the only pentachord to be highlighted within the processes for this piece.

11. Hexachord 6-7* [6] (012678) <420243>: Messiaen Mode V

    i. Hexachord 6-7 is Messiaen's Mode V (literal spelling: 1-1-4-1-1-4)
    ii. Hexachord 6-7 (012678) is a self-contained palindrome (all combinatorial)

12. All-Triad Hexachords (ATH) 6-z17 (012478) and 6-z17B (014678) <322332>:

    i. All-Triad Hexachords (ATH) are named for the fact they contain all twelve prime-form trichords (without replication) within the span of an octave:
        a. 3-1* (012) <210000> BACH/Chromatic Trimirror
        b. 3-2 (013) <111000> Phrygian Trichord
        c. 3-3 (014) <101100> Major-minor Trichord
        d. 3-4 (015) <100110> Incomplete Major Seventh Chord
        e. 3-5 (016) <100011> Rite Chord/Tritone Fourth
        f. 3-6* (024) <020100> Whole-tone Trichord
        g. 3-7 (025) <011010> Incomplete minor seventh Chord
        h. 3-8 (026) <010101> Incomplete Dominant Seventh Chord – Italian Sixth
        i. 3-9* (027) <010020> Quartal Trichord
        j. 3-10* (036) <002001> Diminished Chord
        k. 3-11 (037) <001110> Major/minor Chord (047)
        l. 3-12* [4] (048) <000300> Augmented Chord
    ii. The ATH are self-contained palindromes (combinatorial).

iii. The ATH are subsumed in prime-form within the Symmetrically Inverted All-Interval Twelve-Note (SI AITN) Chords chosen for this piece: 1, 3, 58 & 60.
iv. The SI AITN chords were the theoretical starting point for this research. They offered a flexible way to conceive the largest compositional units (12-tone scales) as symmetrical pitch palindromes on vertical and linear axis.
v. There are many SI AITN to choose from[2], but only four subsume the ATH.
vi. The ATH were therefore fundamental to the cohesive evolution of this research, providing a conceptual central pivot (a conceptual palindrome) to pitch organisation and establishing tangible connections between the largest (SI AITN) and smallest (intervals) compositional units (AIT): Palindromes within palindromes, shapes within shapes (Matroyshka Principle):
vii. SI AITN Chords 1, 3, 58 and 60 all subsume the ATH in prime-form.
viii. The ATH in turn subsume the All-Interval Tetrachords (AIT) in prime-form:
   a. 0137 and 0256 are subsumed within 6-z17 (012478)
   b. 0146 and 0467 are subsumed within 6-z17B (014678)
ix. The AIT in turn subsumes all interval-types and classes.
x. *Rotations and Resonances* for String Quartet (2012) explores the ATH as a primary resource for pitch organisation (consult research).

13. Hexachord 6-14 (013458/034578) <323430>: Parent chord for SI AITN 1 and 58

i. Hexachord 6-14 is the parent hexachord for SI AITN 1 and SI AITN 58.
ii. Hexachord 6-14 is a fundamental unit of the secondary family of resources.
iii. Hexachord 6-14 (self contained palindrome) has a twin cell: 6-14B (034578).

14. Hexachord 6-32* (024579) <143250> Arezzo Major Diatonic (literal spelling: 2-2-1-2-2-3), Major Hexamirror, Quartal Hexamirror: Parent hexachord for SI AITN 60

i. Hexachord 6-32 is the parent hexachord for SI AITN 60.
ii. With its literal spelling of C-D-E-F-G-A, Hexachord 6-32 has obvious tonal/modal foundations. 6-32 belongs to the primary family of resources.
iii. 6-32 can be manipulated to suggest any of the seven modes of Major scale.
iv. Hexachord 6-32 subsumes Pentachord 5-35 (02479) Major Pentatonic Scale
v. Hexachord 6-32 (024579) is a self-contained palindrome (combinatorial).
vi. Hexachord 6-32 is a fundamental unit of the primary family of resources.

15. Hexachord 6-35* [2] (02468T) <060603> Whole-tone Scale: Messiaen Mode I

i. Hexachord 6-35 is Messiaen Mode I – Whole-tone scale (2-2-2-2-2-2).
ii. Hexachord 6-35 is a self-contained palindrome (all-combinatorial).
iii. *A Quartet of Daydreams* for string quartet (2005/09) explores harmonic and melodic characteristics of whole-tone scale and palindromic pitch-wedges.

[2] Carter lists 88, but does not include inversions. Researchers have identified further alternates [Link Chords]

16. Dorian minor Mode – Heptachord 7-35* (013568T) <254361>: Modal Palette

    i. Major Diatonic Heptachord/Dominant 13th, Locrian (1-2-2-1-2-2-2), Phrygian (1-2-2-2-1-2-2) and Major Inverse.
    ii. The Dorian Mode is unique as it is the only mode in the tonal system to invert to itself (also produces identical pitches). It is a natural palindrome.
    iii. The complement to 7-35 (Pentachord 5-35) is a Major Pentatonic Scale, which also inverts to produce the same scale-type (alternating modulation).
    iv. Dorian Mode (7-35) is only Heptachord highlighted within these processes.
    v. Dorian Mode 7-35 (013568T), Hexachord 6-32 (024579), Pentachord 5-35 (02479), Diminished 7th Chord 4-28 (0369) and the Augmented Triad 3-12 (048) are all clearly of tonal/modal foundations: A tonal family adding to the flexibility of available options within the palindromic pitch-palette.
    vi. The AIT can also be interpreted within a modal/tonal setting (see *Ensuen*).

17. Octachord 8-9* [6] (01236789) <644464>: Messiaen Mode IV

    i. Octachord 8-9 is Messiaen Mode IV (literal spelling: 1-1-1-3-1-1-1-3)
    ii. Octachord 8-9 is a self-contained palindrome.

18. Octachord 8-25* [6] (0124678T) <464644>: Messiaen Mode VI

    i. Octachord 8-25 is Messiaen Mode VI (literal spelling: 1-1-2-2-1-1-2-2)
    ii. Octachord 8-25 is a self-contained palindrome.

19. Octachord 8-28* [3] (0134679T) <448444>: Messiaen Mode II

    i. Octachord 8-28 is Messiaen Mode II (literal spelling: 1-2-1-2-1-2-1-2)
    ii. Octachord 8-28 is also known as the alternating octatonic or diminished scale (auxiliary diminished scale).
    iii. Octachord 8-28 is a self-contained palindrome.

20. Nonachord 9-12* [4] (01245689T) <666963>: Messiaen Mode III

    i. Nonachord 9-12 is Messiaen Mode III (literal spelling: 1-1-2-1-1-2-1-1-2)
    ii. Nonachord 9-12 is only Nonachord highlighted within these processes.
    iii. Nonachord 9-12 is a self-contained palindrome.

21. Decachord 10-6* [6] (012346789T) <888885>: Messiaen Mode VII

    i. Decachord 10-6 is Messiaen Mode VII (literal spelling: 1-1-1-1-2-1-1-1-1-2).
    ii. Decachord 10-6 is the only Decachord highlighted within these processes.
    iii. Decachord 10-6 is a self-contained palindrome.

22. All-Interval Twelve-Note (AITN) Chords: Rationale and Choice: 1, 58 and 60

    i. The AITN produce symmetrical (SI AITN) or parallel (PI AITN) inverted, twelve-tone palindromes in vertical chords or linear rows.
    ii. SI AITN chords were the theoretical starting point for this research. They offered a flexible way to conceive the largest compositional units (12-tone scales) as symmetrical pitch palindromes to work on vertical and linear axis.
    iii. AITN use the tritone as the central interval (axis) to the inverted palindrome.
    iv. There are many SI AITN to choose from, but only four subsume the ATH.[3]
    v. SI AITN chords 1, 3, 58 and 60 subsume the ATH in prime-form.
    vi. SI AITN 1 and 58 share the same parent hexachord: 6-14, therefore SI AITN 58 is redundant (though could be used as substitute material for SI AITN 1).
    vii. SI AITN 3 has the parent hexachord 6-1. The total chromatic nature of this hexachord (spans a chromatic tritone) seemed an unsuitable option for this piece. Therefore SI AITN 3 and Hexachord 6-1 are not used in this piece.
    viii. SI AITN 60 has the parent hexachord of 6-32 (024579), this hexachord has blatant foundations in tonality/modality and is the favoured choice for primary material through which to compose a twelve-tone language.
        a. SI AITN 60 (parent chord 6-32) is Primary Twelve-tone material.
        b. SI AITN 1 (parent chord 6-14) is Secondary Twelve-tone material.
        c. SI AITN 58 could be used as a substitute for SI AITN 1 material.
    ix. *Sacsayhuaman* for Symphony Orchestra (2016 – Work in Progress) explores the concept of acoustic saturation and non-linear resonance. These concepts will be interpreted for the orchestral setting through the initial use of SI AITN Chords (consult research) within an opening crescendo (tutti acoustic saturation).

23. Palindromes within Palindromes – Shapes within Shapes – Patterns within Patterns (Matroyshka Principle): Invisible joins …

    i. SI AITN Chords offer the most flexible way of composing symmetrical twelve-tone palindromes (largest pitch palindromes).
    ii. SI AITN Chords 1, 3, 58 and 60 subsume the All-Triad Hexachord (ATH) in prime-form.
    iii. The ATH is the smallest pitch-class set and the most succinct way through which to reference all trichord types highlighted within these processes.
    iv. The ATH subsume the All-Interval Tetrachords (AIT) in prime-form.
    v. The AIT is the smallest pitch-class set and the most succinct way through which to reference all interval-types (smallest pitch palindromes)
    vi. The interval (two notes) and the single pitch are obviously the lowest common denominators of this process (micro palindromes).

[3] Carter lists 88, but does not include inversions. Researchers have identified further alternates [Link Chords]

vii. All compositional units sit within each other in clear parallels to the Matroyshka Principle, however the blatant and recognisable repetition of duplication, imitation, augmentation and diminution of the score contours can be avoided if and when required.
viii. The pitch-palette is cohesively composed from micro through to the macro.
ix. *A Chance Encounter* (Acousmatic) for multiple channel surround sound explores the Matroyshka Principle as a method for flexible 'concrete' form and proportion in acousmatic composition.
x. *An Acoustic Mandala for the Fourteenth* for Chamber Orchestra and Percussion Ensemble (2008/10) explores symmetrical and asymmetrical patterns within patterns as the primary consideration for pitch organisation and refers to external concepts of geometry (shapes within shapes) for musical form and proportion. The orchestral work also explores the concepts of equidistant scales and chromatic consonance (cycle of fourths etc.).

24. Palindromes can be Symmetrical, Parallel, Duplicates or Asymmetrical:

    i. It is well documented that Leonardo Da Vinci often used mirror-writing in his research, but he also employed mirror reflections (parallels) and mirror opposites (inversions) within his art. Researchers highlight how hand gestures often define a point of axis for the mirror, and through this, Da Vinci employed alternate reflections (angles and planes) and mirror imitations that produced asymmetrical (visual) palindromes.

25. Palindromes can be Mirror Opposites, Mirror Reflections, Mirror Inversions and Mirror Retrogrades.

    i. Refer to pitch matrices (below) for further study on the symmetrical and asymmetrical properties for each compositional unit listed.
    ii. Refer to pitch-class flow charts for tangible connections between materials and options for pitch modulation.

26. Palindromes can be Panoptic (all axis), Linear (melodic) and Vertical (harmonic).

27. Palindromes can be Audible, Literal, Theoretical or Subliminal.

All-Interval Twelve-Note Chords

Elliot Carter's All-Interval Twelve-Note (AITN) Chords served as the conceptual genesis for this palindromic pitch organisation. There are two types of AITN Chords: Parallel (PI AITN) and Symmetrically (SI AITN) Inverted. The symmetrically inverted version was chosen as the primary material for this piece because it created a more perfect palindrome in score form (melodic and harmonic contours).

Carter uses four Hexachord (6-note chord) types as the parent chords for his SI AITN:

Hexachord 6-1 (012345) <543210> (Carter 4): Chromatic Hexamirror/1st ord. All combinatorial (P6, Ib, RI5)

Hexachord 6-8 (023457) <343230> (Carter 5): 1st ord. All combinatorial (P6, Ib, RI5)

Hexachord 6-14 (013458) <323430> (Carter 3): Combinatorial P (6) ***
Hexachord 6-14b (034578) <323430> Combinatorial P (6) (6-14 is the prime form) ***

Hexachord 6-32* (024579) <143250> (Carter 6): Arezzo major Diatonic (2-2-1-2-2-3), major hexamirror, quartal hexamirror, 1st ord. All-combinatorial P (6), I (3), RI (9) ***

Note:

i. Hexachord 6-1* (012345) is a chromatic tritone. The total dissonance is not a suitable foundation for this piece: Do NOT use Hexachord 6-1.
ii. Hexachord 6-32* (024579) has tonal/modal origins. This is in keeping with previous research into equidistant and consonant chromatic pitch organisation and would make a good foundation from which to build a twelve-tone language for this piece. Hexachord 6-32 is therefore a primary resource within this piece.
iii. Hexachord 6-14 (013458/034578)) by deduction therefore belongs to the secondary family of resources for this piece.
iv. The Complement and Inversion of Hexachords 6-1, 6-14 and 6-32 produces the same Hexachords: They are all palindromes (combinatorial).

All PI and SI AITN Chords create pitch palindromes, but SI AITN Chords 1, 3, 58 and 60 are the only SI AITN to subsume the All-Triad Hexachord (ATH) in prime form:

ATH 6-z17/6-z43b (012478) <322332> Complement: 6-z17B/6-z43 (014678) <322332>

SI AITN 1. 27431 6 E985T/27431 6 E985T (5-23) (6-14)
SI AITN 3. 27491 6 E 385T/27491 6 E385T (5-35) (6-1)
SI AITN58. 21497 6 5 38ET/21497 6 538ET (5-2) (6-14)
SI AITN60. 21437 6 5 98ET/21437 6 598ET (5-1) (6-32)

Note:

i. As SI AITN 3 is conceived from the parent Hexachord 6-1 (equidistant total dissonance), SI AITN 3 was NOT chosen for this piece: Do NOT use SI AITN 3.
ii. SI AITN 60 has the parent Hexachord 6-32* (024579), which has modal/tonal foundations that makes it an ideal vehicle through which to compose the primary twelve-tone language and establish a suitable balance of dissonance and consonance within the row.
iii. SI AITN 60 and Hexachord 6-32 were therefore selected as the primary family of resources for this piece.
iv. SI AITN 1 and 58 share the same parent Hexachord 6-14 (013458) and therefore share a palindromic relationship in their construction. SI AITN 1 and 58 should be treated as a pair (alternate/substitute).
v. SI AITN 1/58 and Hexachord 6-14 were selected as the secondary family of resources for this piece.

Brief Explanation of AITN Theory:

The lowest pitch of the AITN chord is always considered as 0. One can also use AITN in prime inversion (not listed by Carter), which essentially flips the palindrome and doubles its length. In SI AITN 1 (below), the 1st interval is 2 semi-tones; the 2nd interval is 7 semi-tones and the 3rd interval is 4. If we start with the pitch of C then we have: C – D – A – C#

SI AITN 1: 274316E985T (6-14)
SI AITN 3: 274916E385T (6-1)
SI AITN58: 214976538ET (6-14)
SI AITN60: 214376598ET (6-32)

SI AITN 1: T589E613472 (6-14)
SI AITN 3: T583E619472 (6-1)
SI AITN58: TE835679412 (6-14)
SI AITN60: TE895673412 (6-32)

The AITN always pivots around the tritone interval (interval class 6 inverts to class 6)

SI AITN 1: 27431 6 E985T (6-14)
SI AITN 3: 27491 6 E385T (6-1)
SI AITN58: 21497 6 538ET (6-14)
SI AITN60: 21437 6 598ET (6-32)

SI AITN 1: T589E 6 13472 (6-14)
SI AITN 3: T583E 6 19472 (6-1)
SI AITN58: TE835 6 79412 (6-14)
SI AITN60: TE895 6 73412 (6-32)

AITN 1, 3, 58 and 60 contain the All-Triad Hexachord (ATH): 012478/014678

SI AITN 1: 27431 6 E985T/27431 6 E985T (6-14) [Secondary Material]
SI AITN 3: 27491 6 E385T/27491 6 E385T (6-1) [Do not use this AITN] ***
SI AITN58: 21497 6 538ET/21497 6 538ET (6-14) [Alternate for SI AITN 1]
SI AITN60: 21437 6 598ET/21437 6 598ET (6-32) [Primary Material]

Pitch Matrices

4-z15: (0146/0256) <111111> All-Interval Tetrachord (AIT) 1 and 2 (Inverts to 4-z15)

0	1	4	6
11	0	3	5
8	9	0	2
6	7	10	0

C	$C^{\#}$	E	$F^{\#}$
B	C	E^{b}	F
$G^{\#}$	A	C	D
$F^{\#}$	G	B^{b}	C

4-z29: (0137/0467) <111111> All-Interval Tetrachord (AIT) 3 and 4 (Inverts to 4-z29)

0	1	3	7
11	0	2	6
9	10	0	4
5	6	8	0

C	D^{b}	E^{b}	G
B	C	D	$F^{\#}$
A	B^{b}	C	E
F	G^{b}	A^{b}	C

4-9* [6]: (0167) <200022> Double Tritone Tetramirror: Messiaen Mode IV Complement

0	1	6	7
11	0	5	6
6	7	0	1
5	6	11	0

C	$C^{\#}$	$F^{\#}$	G
B	C	G	$F^{\#}$
$F^{\#}$	G	C	D^{b}
F	$F^{\#}$	B	C

4-25* [6]: (0268) <020202> French Sixth Chord: Mode VI Complement

0	2	6	8
10	0	4	6
6	8	0	2
4	6	10	0

C	D	$F^{\#}$	$G^{\#}$
$A^{\#}$	C	E	$F^{\#}$
$F^{\#}$	$G^{\#}$	C	D
E	$F^{\#}$	$A^{\#}$	C

4-28* [3]: (0369) <004002> Diminished Seventh Chord: Messiaen Mode II Complement

0	3	6	9
9	0	3	6
6	9	0	3
3	6	9	0

C	$D^{\#}$	$F^{\#}$	A
A	C	$D^{\#}$	$F^{\#}$
$F^{\#}$	A	C	$D^{\#}$
$D^{\#}$	$F^{\#}$	A	C

Note: 4-28 (0369) not subsumed within any larger set listed (smaller than an octachord). 4-26 could act as a neutral transition or could be part of the primary modal/tonal family.

5-35*: (02479) <032140> Major Pentatonic Scale (2-2-3-2-3): Black Key Pentatonic/Quartal Pentamirror: Dorian Mode Complement and subsumed within 6-32

0	2	4	7	9
10	0	2	5	7
8	10	0	3	5
5	7	9	0	2
3	5	7	10	0

C	D	E	G	A
B^b	C	D	F	G
A^b	B^b	C	E^b	F
F	G	A	C	D
E^b	F	G	B^b	C

Note: 5-35 (02479) has tonal/modal origins & belongs to the primary family of resources.

6-z17: (012478/014678) <322332> All-Triad Hexachord (ATH): Inverts to 6-z17
012478 subsume 0137&0256 [1248=0137, 2478=0256], 014678 subsume 0146&0467

0	1	2	4	7	8
11	0	1	3	6	7
10	11	0	2	5	6
8	9	10	0	3	4
5	6	7	9	0	1
4	5	6	8	11	0

C	$C^\#$	D	E	G	A^b
B	C	$C^\#$	E^b	$F^\#$	G
B^b	B	C	D	F	$F^\#$
A^b	A	B^b	C	E^b	E
F	$F^\#$	G	A	C	$C^\#$
E	F	$F^\#$	A^b	B	C

6-14: (013458/034578) <323430> Combinatorial P (6): Inversion & Complement = 6-14

0	1	3	4	5	8
11	0	2	3	4	7
9	10	0	1	2	5
8	9	11	0	1	4
7	8	10	11	0	3
4	5	7	8	9	0

C	$C^\#$	$D^\#$	E	F	$G^\#$
B	C	D	$D^\#$	E	G
A	B^b	C	D^b	D	F
$G^\#$	A	B	C	$C^\#$	E
G	A^b	B^b	B	C	E^b
E	F	G	$G^\#$	A	C

Note: 6-14 (013458) is parent for SI AITN 1 & 58 and belongs to secondary resources.

6-32*: (024579) <143250> Arezzo major Diatonic (2-2-1-2-2-3), major hexamirror, quartal hexamirror, 1st ord. All-combinatorial P (6), I (3), RI (9): Modal/Tonal Family

0	2	4	5	7	9
10	0	2	3	5	7
8	10	0	1	3	5
7	9	11	0	2	4
5	7	9	10	0	2
3	5	7	8	10	0

C	D	E	F	G	A
B^b	C	D	E^b	F	G
A^b	B^b	C	D^b	E^b	F
G	A	B	C	D	E
F	G	A	B^b	C	D
E^b	F	G	A^b	B^b	C

Note: 6-32* (024579) is parent for SI AITN 60 and belongs to primary resources.

7-35*: (013568T) <254361> Dorian minor Mode:
Only mode to invert to itself (unique in the modal/tonal system)
Spelling: 2-1-2-2-2-1-2 (C-D-E^b-F-G-A-B^b-C)
Heptachord 7-35* (013568T) <254361> Palindrome: Inverts to 7-35
Complement: 5-35 (02479) Major Pentatonic Scale <032140> Palindrome: Inverts to 5-35

0	2	3	5	7	9	10	0
10	0	1	3	5	7	8	10
9	11	0	2	4	6	7	9
7	9	10	0	2	4	5	7
5	7	8	10	0	2	3	5
3	5	6	8	10	0	1	3
2	4	5	7	9	11	0	2
0	2	3	5	7	9	10	0

C	D	E^b	F	G	A	B^b	C
B^b	C	D^b	E^b	F	G	A^b	B^b
A	B	C	D	E	$F^\#$	G	A
G	A	B^b	C	D	E	F	G
F	G	A^b	B^b	C	D	E^b	F
E^b	F	G^b	A^b	B^b	C	D^b	E^b
D	E	F	G	A	B	C	D
C	D	E^b	F	G	A	B^b	C

Note: Strong references to the Dorian mode are present within SI AITN 60 (lower hexachord = C Dorian, upper hexachord = B Dorian) and the Hexachord 6-32 (024579). SI AITN 60 also passes through a semi-tone modulation from C Dorian – B Dorian (0 = C).

Note: Dorian Mode (7-35) is related to SI AITN 60 and Hexachord 6-32, it is therefore part of both the primary family of resources and the tonal/modal family of resources.

SI AITN 1: 27431 6 E985T – 27431 6 E985T (6-14): Subsumes 6-Z17 twice
Literal Spelling: 029145 ET7386 = C – D – A – $C^\#$ – E – F – B – $A^\#$ – G – $D^\#$ – $G^\#$ – $F^\#$
Literal Spelling: 029145 ET7386 = C – D – A – $C^\#$ – E – F – B – $A^\#$ – G – $D^\#$ – $G^\#$ – $F^\#$

0	2	9	1	4	5	11	10	7	3	8	6
10	0	7	11	2	3	9	8	5	1	6	4
3	5	0	4	7	8	2	1	10	6	11	9
11	1	8	0	3	4	10	9	6	2	7	5
8	10	5	9	0	1	7	6	3	11	4	2
7	9	4	8	11	0	6	5	2	10	3	1
1	3	10	2	5	6	0	11	8	4	9	7
2	4	11	3	6	7	1	0	9	5	10	8
5	7	2	6	9	10	4	3	0	8	1	11
9	11	6	10	1	2	8	7	4	0	5	3
4	6	1	5	8	9	3	2	11	7	0	10
6	8	3	7	10	11	5	4	1	9	2	0

Note: SI AITN 1 and 58 share the same parent Hexachord 6-14 (013458). They are part of the secondary family of resources and should be treated as an alternate pair.

SI AITN 3: 27491 6 E385T/27491 6 E385T (6-1) Subsumes 6-Z17 twice [Do not use]**
Literal Spelling: 0291TE 547386 = C – D – A – $C^{\#}$ – $A^{\#}$ – B – F – E – G – $D^{\#}$ – $G^{\#}$ – $F^{\#}$
Literal Spelling: 0291TE 547386 = C – D – A – $C^{\#}$ – $A^{\#}$ – B – F – E – G – $D^{\#}$ – $G^{\#}$ – $F^{\#}$

SI AITN58: 21497 6 538ET – 21497 6 538ET (6-14) Subsumes 6-Z17 twice
Literal Spelling: 02374E 5T1986 = C – D – $D^{\#}$ – G – E – B – F – B^{b} – D^{b} – A – $G^{\#}$ – $F^{\#}$
Literal Spelling: 02374E 5T1986 = C – D – $D^{\#}$ – G – E – B – F – B^{b} – D^{b} – A – $G^{\#}$ – $F^{\#}$

0	2	3	7	4	11	5	10	1	9	8	6
10	0	1	5	2	9	3	8	11	7	6	4
9	11	0	4	1	8	2	7	10	6	5	3
5	7	8	0	9	4	10	3	6	2	1	11
8	10	11	3	0	7	1	6	9	5	4	2
1	3	4	8	5	0	6	11	2	10	9	7
7	9	10	2	11	6	0	5	8	4	3	1
2	4	5	9	6	1	7	0	3	11	10	8
11	1	2	6	3	10	4	9	0	8	7	5
3	5	6	10	7	2	8	1	4	0	11	9
4	6	7	11	8	3	9	2	5	1	0	10
6	8	9	1	10	5	11	4	7	3	2	0

Note: SI AITN 1 and 58 share the same parent Hexachord 6-14 (013458). They are part of the secondary family of resources and should be treated as an alternate pair.

SI AITN60: 21437 6 598ET – 21437 6 598ET (6-32) Subsumes 012478 twice
Literal Spelling: 0237T5 E41986 = C – D – E^{b} – G – B^{b} – F – B – E – $C^{\#}$ – A – $G^{\#}$ – $F^{\#}$
Literal Spelling: 0237T5 E41986 = C – D – E^{b} – G – B^{b} – F – B – E – $C^{\#}$ – A – $G^{\#}$ – $F^{\#}$

0	2	3	7	10	5	11	4	1	9	8	6
10	0	1	5	8	3	9	2	11	7	6	4
9	11	0	4	7	2	8	1	10	6	5	3
5	7	8	0	3	10	4	9	6	2	1	11
2	4	5	9	0	7	1	6	3	11	10	8
7	9	10	2	5	0	6	11	8	4	3	1
1	3	4	8	11	6	0	5	2	10	9	7
8	10	11	3	6	1	7	0	9	5	4	2
11	1	2	6	9	4	10	3	0	8	7	5
3	5	6	10	1	8	2	7	4	0	11	9
4	6	7	11	2	9	3	8	5	1	0	10
6	8	9	1	4	11	5	10	7	3	2	0

Note: SI AITN 60 belongs to the primary family of resources for this piece. Parent Hexachord 6-32 (024579) has modal/tonal origins and is closely related to 5-35 and 7-35.

Modes of Limited Transposition

Olivier Messiaen

Messiaen Mode I (Whole-tone Scale): Two transpositions (two modes)
Literal Spelling: 2-2-2-2-2-2 (C-D-E-F$^{\#}$-G$^{\#}$-A$^{\#}$-C)
Hexachord 6-35* [2] (02468T) <060603> Palindrome: Inverts to 6-35: Complement: 6-35

Messiaen Mode II (Octatonic or Diminished Scale): Three transpositions (three modes)
Literal Spelling: 1-2-1-2-1-2-1-2 (auxiliary diminished) OR 2-1-2-1-2-1-2-1 (diminished)
Octachord 8-28* [3] (0134679T) <448444> Palindrome: Inverts to 8-28
Complement: 4-28* [3] (0369) Diminished 7th Chord <004002> Inverts to 4-28
Note: Subsumes AIT: 0137/0467 - 0146/0256 [1367 = 0256]

Messiaen Mode III: Four transpositions (four modes)
Literal Spelling: 2-1-1-2-1-1-2-1-1 (C-D-E^{b}-E-F$^{\#}$-G-A-B^{b}-B-C)
Nonachord 9-12* [4] (01245689T) <666963> Palindrome: Inverts to 9-12
Complement: 3-12* [4] (048) Augmented Triad <000300> Palindrome: Inverts to 3-12

Messiaen Mode IV: Six transpositions (six modes)
Literal Spelling: 1-1-1-3-1-1-1-3 (C-D^{b}-D-E^{b}-G^{b}-G-A^{b}-A-C)
Octachord 8-9* [6] (01236789) <644464> Palindrome: Inverts to 8-9
Complement: Tetrachord 4-9 (0167) <200022> Palindrome: Inverts to 4-9

Messiaen Mode V: Six transpositions (six modes)
Literal Spelling: 1-1-4-1-1-4 (C-D^{b}-D-F$^{\#}$-G-A^{b}-C)
Hexachord 6-7* [6] (012678) <420243> Palindrome: Inverts to 6-7: Complement: 6-7

Messiaen Mode VI: Six transpositions (six modes)
Literal Spelling: 1-1-2-2-1-1-2-2 (C-D^{b}-D-E-F$^{\#}$-G-A^{b}-B^{b}-C)
Octachord 8-25* [6] (0124678T) <464644> Palindrome: Inverts to 8-25
Complement: 4-25* [6] (0268) French Sixth Chord <020202> Palindrome: Inverts to 4-25

Messiaen Mode VII: Six transpositions (six modes)
Literal spelling: 1-1-1-1-2-1-1-1-1-2 (C-D^{b}-D-E^{b}-E-F$^{\#}$-G-G$^{\#}$-A-B^{b}-C)
Decachord 10-6* [6] (012346789T) <888885> Palindrome: Inverts to 10-6
Complement: Tritone 2-6* [6] (06) <000001> Palindrome: Tritone inverts to tritone

Messiaen Modes Pitch Matrices

6-35*: Messiaen Mode I (Whole-tone Scale): Two transpositions (two modes)
Literal Spelling: 2-2-2-2-2-2 (C-D-E-F$^{\#}$-G$^{\#}$-A$^{\#}$-C)
Hexachord 6-35* [2] (02468T) <060603> Palindrome: Inverts to 6-35: Complement: 6-35

0	2	4	6	8	10
10	0	2	4	6	8
8	10	0	2	4	6
6	8	10	0	2	4
4	6	8	10	0	2
2	4	6	8	10	0

C	D	E	F$^{\#}$	G$^{\#}$	B^{b}
B^{b}	C	D	E	F$^{\#}$	G$^{\#}$
A^{b}	B^{b}	C	D	E	F$^{\#}$
G^{b}	A^{b}	B^{b}	C	D	E
E	G^{b}	A^{b}	B^{b}	C	D
D	E	G^{b}	A^{b}	B^{b}	C

8-28*: Mode II (Octatonic or Diminished Scale): Three transpositions (three modes)
Literal Spelling: 1-2-1-2-1-2-1-2 (auxiliary diminished) OR 2-1-2-1-2-1-2-1 (diminished)
Octachord 8-28* [3] (0134679T) <448444> Palindrome: Inverts to 8-28
Complement: 4-28* [3] (0369) Diminished 7th Chord <004002> Inverts to 4-28
Note: Subsumes AIT: 0137/0467 - 0146/0256 [1367 = 0256]

0	1	3	4	6	7	9	10
11	0	2	3	5	6	8	9
9	10	0	1	3	4	6	7
8	9	11	0	2	3	5	6
6	7	9	10	0	1	3	4
5	6	8	9	11	0	2	3
3	4	6	7	9	10	0	1
2	3	5	6	8	9	11	0

C	C$^{\#}$	D$^{\#}$	E	F$^{\#}$	G	A	B^{b}
B	C	D	E^{b}	F	F$^{\#}$	G$^{\#}$	A
A	B^{b}	C	D^{b}	E^{b}	E	F$^{\#}$	G
G$^{\#}$	A	B^{b}	C	D	E^{b}	F	F$^{\#}$
F$^{\#}$	G	A	B^{b}	C	D^{b}	E^{b}	E
F	F$^{\#}$	G$^{\#}$	A	B	C	D	E^{b}
E^{b}	E	F$^{\#}$	G	A	B^{b}	C	D^{b}
D	E^{b}	F	F$^{\#}$	G$^{\#}$	A	B	C

9-12*: Messiaen Mode III: Four transpositions (four modes)
Literal Spelling: 2-1-1-2-1-1-2-1-1 (C-D-E^{b}-E-F$^{\#}$-G-A-B^{b}-B-C)
Nonachord 9-12* [4] (01245689T) <666963> Palindrome: Inverts to 9-12
Complement: 3-12* [4] (048) Augmented Triad <000300> Palindrome: Inverts to 3-12

0	1	2	4	5	6	8	9	10
11	0	1	3	4	5	7	8	9
10	11	0	2	3	4	6	7	8
8	9	10	0	1	2	4	5	6
7	8	9	11	0	1	3	4	5
6	7	8	10	11	0	2	3	4
4	5	6	8	9	10	0	1	2
3	4	5	7	8	9	11	0	1
2	3	4	6	7	8	10	11	0

8-9*: Messiaen Mode IV: Six transpositions (six modes)
Literal Spelling: 1-1-1-3-1-1-1-3 (C-D^{b}-D-E^{b}-G^{b}-G-A^{b}-A-C)
Octachord 8-9* [6] (01236789) <644464> Palindrome: Inverts to 8-9
Complement: Tetrachord 4-9 (0167) <200022> Palindrome: Inverts to 4-9

0	1	2	3	6	7	8	9
11	0	1	2	5	6	7	8
10	11	0	1	4	5	6	7
9	10	11	0	3	4	5	6
6	7	8	9	0	1	2	3
5	6	7	8	11	0	1	2
4	5	6	7	10	11	0	1
3	4	5	6	9	10	11	0

C	C$^{\#}$	D	D$^{\#}$	F$^{\#}$	G	G$^{\#}$	A
B	C	C$^{\#}$	D	F	F$^{\#}$	G	G$^{\#}$
B^{b}	B	C	C$^{\#}$	E	F	F$^{\#}$	G
A	B^{b}	B	C	D$^{\#}$	E	F	F$^{\#}$
F$^{\#}$	G	G$^{\#}$	A	C	C$^{\#}$	D	D$^{\#}$
F	F$^{\#}$	G	G$^{\#}$	B	C	C$^{\#}$	D
E	F	F$^{\#}$	G	B^{b}	B	C	C$^{\#}$
D$^{\#}$	E	F	F$^{\#}$	A	B^{b}	B	C

6-7*: Messiaen Mode V: Six transpositions (six modes)
Literal Spelling: 1-1-4-1-1-4 (C-D^{b}-D-F$^{\#}$-G-A^{b}-C)
Hexachord 6-7* [6] (012678) <420243> Palindrome: Inverts to 6-7: Complement: 6-7

0	1	2	6	7	8
11	0	1	5	6	7
10	11	0	4	5	6
6	7	8	0	1	2
5	6	7	11	0	1
4	5	6	10	11	0

C	C$^{\#}$	D	F$^{\#}$	G	G$^{\#}$
B	C	C$^{\#}$	F	F$^{\#}$	G
B^{b}	B	C	E	F	F$^{\#}$
F$^{\#}$	G	G$^{\#}$	C	C$^{\#}$	D
F	F$^{\#}$	G	B	C	C$^{\#}$
E	F	F$^{\#}$	A$^{\#}$	B	C

8-25* Messiaen Mode VI: Six transpositions (six modes)
Literal Spelling: 1-1-2-2-1-1-2-2 (C-D^{b}-D-E-F$^{\#}$-G-A^{b}-B^{b}-C)
Octachord 8-25* [6] (0124678T) <464644> Palindrome: Inverts to 8-25
Complement: 4-25* [6] (0268) French Sixth Chord <020202> Palindrome: Inverts to 4-25

0	1	2	4	6	7	8	10
11	0	1	3	5	6	7	9
10	11	0	2	4	5	6	8
8	9	10	0	2	3	4	6
6	7	8	10	0	1	2	4
5	6	7	9	11	0	1	3
4	5	6	8	10	11	0	2
2	3	4	6	8	9	10	0

C	C$^{\#}$	D	E	F$^{\#}$	G	G$^{\#}$	B^{b}
B	C	C$^{\#}$	D$^{\#}$	F	F$^{\#}$	G	A
B^{b}	B	C	D	E	F	F$^{\#}$	G$^{\#}$
G$^{\#}$	A	B^{b}	C	D	D$^{\#}$	E	F$^{\#}$
F$^{\#}$	G	G$^{\#}$	B^{b}	C	C$^{\#}$	D	E
F	F$^{\#}$	G	A	B	C	C$^{\#}$	D$^{\#}$
E	F	F$^{\#}$	G$^{\#}$	B^{b}	B	C	D
D	D$^{\#}$	E	F$^{\#}$	G$^{\#}$	A	B^{b}	C

Note: The matrix for Mode VI is not as symmetrical as it should be – there seems to be an anomaly here for potential exploration/investigation …

10-6*: Messiaen Mode VII: Six transpositions (six modes)

Literal spelling: 1-1-1-1-2-1-1-1-1-2 (C-D^{b}-D-E^{b}-E-F$^{\#}$-G-G$^{\#}$-A-B^{b}-C)

Decachord 10-6* [6] (012346789T) <888885> Palindrome: Inverts to 10-6
Complement: Tritone 2-6* [6] (06) <000001> Palindrome: Tritone inverts to tritone

0	1	2	3	4	6	7	8	9	10
11	0	1	2	3	5	6	7	8	9
10	11	0	1	2	4	5	6	7	8
9	10	11	0	1	3	4	5	6	7
8	9	10	11	0	2	3	4	5	6
6	7	8	9	10	0	1	2	3	4
5	6	7	8	9	11	0	1	2	3
4	5	6	7	8	10	11	0	1	2
3	4	5	6	7	9	10	11	0	1
2	3	4	5	6	8	9	10	11	0

C	D^{b}	D	E^{b}	E	F$^{\#}$	G	G$^{\#}$	A	B^{b}
B	C	D^{b}	D	E^{b}	F	F$^{\#}$	G	G$^{\#}$	A
B^{b}	B	C	D^{b}	D	E	F	F$^{\#}$	G	G$^{\#}$
A	B^{b}	B	C	D^{b}	E^{b}	E	F	F$^{\#}$	G
G$^{\#}$	A	B^{b}	B	C	D	E^{b}	E	F	F$^{\#}$
F$^{\#}$	G	G$^{\#}$	A	B	C	D^{b}	D	E^{b}	E
F	F$^{\#}$	G	G$^{\#}$	A	B	C	D^{b}	D	E^{b}
E	F	F$^{\#}$	G	G$^{\#}$	B^{b}	B	C	D^{b}	D
E^{b}	E	F	F$^{\#}$	G	A	B^{b}	B	C	D^{b}
D	E^{b}	E	F	F$^{\#}$	G$^{\#}$	A	B^{b}	B	C

List of Pitch-Class Sets

1. Unison (Octave): The details of the individual note.
2. Intervals, Interval-types and Interval-classes
3. Interval Class Five: 2-5 (05) <000010> Perfect Fourth: Cycle of Fourths
4. Interval Class Six: 2-6* [6] (06) <000001> Tritone: Mode VII Complement
5. Trichord 3-12* [4] (048) <000300> Augmented Triad: Mode III Complement
6. Tetrachord 4-9* [6] (0167) <200022> Double Tritone Tetramirror: Messiaen Mode IV Complement
7. All-Interval Tetrachords (AIT):
   i. 4-z15/4-z15B (0146/0256) <111111>
   ii. 4-z29/4-z29B (0137/0467) <111111>
8. Tetrachord 4-25* [6] (0268) <020202> French Sixth Chord: Mode VI Complement
9. Tetrachord 4-28* [3] (0369) <004002> Diminished Seventh Chord – Vagrant Chord Messiaen Mode II Complement: Not subsumed in any set smaller than octachord
10. Pentachord 5-35* (02479) <032140> Major Pentatonic Scale – Black Key Pentatonic/Quartal Pentamirror: Dorian mode complement & subsumed within 6-32
11. Hexachord 6-7* [6] (012678) <420243> Messiaen Mode V
12. Hexachord 6-14 (013458/034578) <323430> Parent chord for SI AITN 1 and 58
13. All-Triad Hexachord (ATH):
   i. 6-z17 (012478) <322332>
   ii. 6-z17B (014678) <322332>
14. Hexachord 6-32* (024579) <143250> Arezzo Major Diatonic (literal spelling: 2-2-1-2-2-3), Major Hexamirror, Quartal Hexamirror: Parent hexachord for SI AITN 60
15. Hexachord 6-35* [2] (02468T) <060603> Whole-tone Scale: Messiaen Mode I
16. Heptachord 7-35* (013568T) <254361> Dorian minor Mode: Modal Palette
17. Octachord 8-9* [6] (01236789) <644464> Messiaen Mode IV
18. Octachord 8-25* [6] (0124678T) <464644> Messiaen Mode VI
19. Octachord 8-28* [3] (0134679T) <448444> Messiaen Mode II
20. Nonachord 9-12* [4] (01245689T) <666963>: Messiaen Mode III
21. Decachord 10-6* [6] (012346789T) <888885>: Messiaen Mode VII
22. SI AITN Chord 1: Secondary Material (alternate for SI AITN 58)
   i. Original: 27431 6 E985T (6-14)
   ii. Inversion: T589E 6 13472 (6-14)
   iii. Literal Spelling: 029145 ET7386 = C-D-A-$C^{\#}$-E-F-B-$A^{\#}$-G-$D^{\#}$-$G^{\#}$-$F^{\#}$
23. SI AITN Chord 58: Secondary Material (alternate for SI AITN 1)
   i. Original: 21497 6 538ET (6-14)
   ii. Inversion: TE835 6 79412 (6-14)
   iii. Literal Spelling: 02374E 5T1986 = C-D-$D^{\#}$-G-E-B-F-B^{b}-D^{b}-A-$G^{\#}$-$F^{\#}$
24. SI AITN Chord 60: Primary Material (modal origins and semi-tone modulation)
   i. Original: 21437 6 598ET (6-32)
   ii. Inversion: TE895 6 73412 (6-32)
   iii. Literal Spelling: 0237T5 E41986 = C-D-E^{b}-G-B^{b}-F-B-E-$C^{\#}$-A-$G^{\#}$-$F^{\#}$

List of Pitch-class Sets
SI AITN Chord 1
Secondary Resources
Literal Spelling: C – D – A – C# – E – F – B – A# – G – D# – G# – F# – C
27431 6 E985T (6-14)

SI AITN 1 – Trichords:

C	D	A	025	3-7	<011010>	Incomplete minor seventh Chord (035)
D	A	C#	015	3-4	<100110>	Incomplete Major Seventh Chord (045)
A	C#	E	037	3-11	<001110>	Major/Minor Chord (047)
C#	E	F	014	3-3	<101100>	Major-minor Trichord 1 (034)
E	F	B	016	3-5	<100011>	Rite Chord/Tritone Fourth (056)
F	B	A#	016	3-5	<100011>	Rite Chord/Tritone Fourth (056)
B	A#	G	014	3-3	<101100>	Major-minor Trichord 1 (034)
A#	G	D#	037	3-11	<001110>	Major/Minor Chord (047)
G	D#	G#	015	3-4	<100110>	Incomplete Major Seventh Chord (045)
D#	G#	F#	025	3-7	<011010>	Incomplete minor seventh Chord (035)
G#	F#	C	026	3-8	<010101>	Incomplete Dominant Seventh/Italian Sixth (046)
F#	C	D	026	3-8	<010101>	Incomplete Dominant Seventh/Italian Sixth (046)
C	D	A	025	3-7	<011010>	Incomplete minor seventh Chord (035)

Note: 3-12 (048) is not subsumed in prime form within any of the SI AITN chords. *

SI AITN 1 – Tetrachords:

C	D	A	C#	0125	4-4	<211110>	Minor Third Tetracluster 2 (0345)
D	A	C#	E	0237	4-14<[4]	<111120>	Major-second Minor Tetrachord (0457)
A	C#	E	F	0148	4-19	<101310>	Minor-augmented Tetrachord (0348)
C#	E	F	B	0146	4-z15	<111111>	All-Interval Tetrachord 1 (0256)
E	F	B	A#	0167	4-9*[6]	<200022>	Double Tritone – Mode IV Complement
F	B	A#	G	0146	4-z15	<111111>	All-Interval Tetrachord 1 (0256)
B	A#	G	D#	0148	4-19	<101310>	Minor-augmented Tetrachord (0348)
A#	G	D#	G#	0237	4-14<	<111120>	Major-second Minor Tetrachord (0457)
G	D#	G#	F#	0125	4-4	<211110>	Minor Third Tetracluster 2 (0345)
D#	G#	F#	C	0258	4-27	<012111>	Half-diminished Seventh Chord (0368)
G#	F#	C	D	0268	4-25*[6]	<020202>	French Sixth – Mode VI Complement
F#	C	D	A	0258	4-27	<012111>	Half-diminished Seventh Chord (0368)
C	D	A	C#	0125	4-4	<211110>	Minor Third Tetracluster 2 (0345)

Note: 4-28 (0369) is not subsumed in prime form within any of the SI AITN Chords. *

[4] < Indicates set has a complement with the same name ending.

SI AITN 1 – Pentachords:

C	D	A	C#	E	02347	5-11	<222220>	Centre-cluster Pentachord 1 (03457)
D	A	C#	E	F	01348	5-z17*	<212320>	Minor-Major Ninth Chord
A	C#	E	F	B	01468	5-30	<121321>	Enigmatic Pentachord 1 (02478)
C#	E	F	B	A#	01367	5-19	<212122>	Javanese Pentachord (01467)
E	F	B	A#	G	01367	5-19	<212122>	Javanese Pentachord (01467)
F	B	A#	G	D#	01468	5-30	<121321>	Enigmatic Pentachord 1 (02478)
B	A#	G	D#	G#	01348	5-z17*	<212320>	Minor-Major Ninth Chord
A#	G	D#	G#	F#	02347	5-11	<222220>	Centre-cluster Pentachord 1 (03457)
G	D#	G#	F#	C	01258	5-z38	<212221>	Diminished Pentacluster 1 (03678)
D#	G#	F#	C	D	02368	5-28<	<122212>	Augmented-sixth Pentachord1 02568
G#	F#	C	D	A	02368	5-28<	<122212>	Augmented-sixth Pentachord1 02568
F#	C	D	A	C#	01258	5-z38	<212221>	Diminished Pentacluster 1 (03678)
C	D	A	C#	E	02347	5-11	<222220>	Centre-cluster Pentachord 1 (03457)

Note: 5-35* (02479) is not subsumed in prime form within any of the SI AITN Chords. *

SI AITN 1 – Hexachords:

C	D	A	C#	E	F	013458	6-14	<323430>	Combinatorial (034578)
D	A	C#	E	F	B	013468	6-z24	<233331>	B: 024578 Melodic minor
A	C#	E	F	B	A#	012478	6-z17	<322332>	All-Triad Hexachord (014678)
C#	E	F	B	A#	G	013679	6-30 [6]	<224223>	Minor-bitonal comb. 023689
E	F	B	A#	G	D#	012478	6-z17	<322332>	All-Triad Hexachord (014678)
F	B	A#	G	D#	G#	013468	6-z24	<233331>	B: 024578 Melodic minor
B	A#	G	D#	G#	F#	013458	6-14	<323430>	Combinatorial (034578)
A#	G	D#	G#	F#	C	012469	6-z46	<233331>	6-z46B: 024569
G	D#	G#	F#	C	D	012568	6-z43	<322332>	6-z43B: 023678
D#	G#	F#	C	D	A	013679	6-30 [6]	<224223>	Minor-bitonal comb. 023689
G#	F#	C	D	A	C#	012568	6-z43	<322332>	6-z43B: 023678
F#	C	D	A	C#	E	012469	6-z46	<233331>	6-z46B: 024569
C	D	A	C#	E	F	013458	6-14	<323430>	Combinatorial (034578)

Note: 6-7* (012678) and 6-35* (02468T) are not subsumed in prime form within any of the SI AITN Chords. *

SI AITN 1 – Heptachords:

C	D	A	C#	E	F	B	0134568	7-11	<444441>	7-11B: 0234578
D	A	C#	E	F	B	A#	0124578	7-z38	<434442>	7-z38B: 0134678
A	C#	E	F	B	A#	G	0135679	7-28<	<344433>	7-28B: 0234689
C#	E	F	B	A#	G	D#	0135679	7-28<	<344433>	7-28B: 0234689
E	F	B	A#	G	D#	G#	0124578	7-z38	<434442>	7-z38B: 0134678
F	B	A#	G	D#	G#	F#	0134568	7-11	<444441>	7-11B: 0234578
B	A#	G	D#	G#	F#	C	0124569	7-z17	<434541>	
A#	G	D#	G#	F#	C	D	0124689 0135789	7-30	<343542>	Neapolitan-Minor (1222131) Mela Dhenuka
G	D#	G#	F#	C	D	A	0123679	7-19	<434343>	7-19B: 0123689
D#	G#	F#	C	D	A	C#	0123679	7-19	<434343>	7-19B: 0123689
G#	F#	C	D	A	C#	E	0124689 0135789	7-30	<343542>	Neapolitan-Minor (1222131) Mela Dhenuka
F#	C	D	A	C#	E	F	0124569	7-z17	<434541>	
C	D	A	C#	E	F	B	0134568	7-11	<444441>	7-11B: 0234578

Note: 7-35* (013568T) is not subsumed in prime form within any of the SI AITN Chords. *

SI AITN 1 – Octachords:

C	D	A	C#	E	F	B	A#	01234578	8-4	<655552>	8-4B: 01345678
D	A	C#	E	F	B	A#	G	0124578T	8-27	<456553>	8-27B: 0124679T
A	C#	E	F	B	A#	G	D#	0124678T	8-25*	<464644>	Mode VI
C#	E	F	B	A#	G	D#	G#	0124578T	8-27	<456553>	8-27B: 0124679T
E	F	B	A#	G	D#	G#	F#	01234578	8-4	<655552>	8-4B: 01345678
F	B	A#	G	D#	G#	F#	C	01245679	8-14<	<555562>	8-14B: 02345789
B	A#	G	D#	G#	F#	C	D	01245689	8-19	<545752>	8-19B: 01345789
A#	G	D#	G#	F#	C	D	A	01234689	8-z15	<555553>	8-z15B: 01356789
G	D#	G#	F#	C	D	A	C#	01236789	8-9	<644464>	Mode IV
D#	G#	F#	C	D	A	C#	E	01234689	8-z15	<555553>	8-z15B: 01356789
G#	F#	C	D	A	C#	E	F	01245689	8-19	<545752>	8-19B: 01345789
F#	C	D	A	C#	E	F	B	01245679	8-14<	<555562>	8-14B: 02345789
C	D	A	C#	E	F	B	A#	01234578	8-4	<655552>	8-4B: 01345678

Note: 8-28* (0134679T) is not subsumed within any of the SI AITN Chords. *

SI AITN 1 – Nonachords:

C	D	A	$C^{\#}$	E	F	B	$A^{\#}$	G	01234578T	9-7	<677673>	B: 01234579T NonatonicBlues
D	A	$C^{\#}$	E	F	B	$A^{\#}$	G	$D^{\#}$	01234678T	9-8	<676764>	B: 01234689T
A	$C^{\#}$	E	F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	01234678T	9-8	<676764>	B: 01234689T
$C^{\#}$	E	F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	01234578T	9-7	<677673>	B: 01234579T NonatonicBlues
E	F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	012345789	9-4	<766773>	B: 012456789
F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	01235679T	9-11	<667773>	B: 01235689T
B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	012345689	9-3	<767763>	B: 013456789
$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	012346789	9-5	<766674>	B: 012356789
G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	E	012346789	9-5	<766674>	B: 012356789
$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	E	F	012345689	9-3	<767763>	B: 013456789
$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	E	F	B	01235679T	9-11	<667773>	B: 01235689T
$F^{\#}$	C	D	A	$C^{\#}$	E	F	B	$A^{\#}$	012345789	9-4	<766773>	B: 012456789
C	D	A	$C^{\#}$	E	F	B	$A^{\#}$	G	01234578T	9-7	<677673>	B: 01234579T NonatonicBlues

Note: 9-12* (01245689T) is not subsumed within any of the SI AITN Chords. *

SI AITN 1 – Decachords:

C	D	A	$C^{\#}$	E	F	B	$A^{\#}$	G	$D^{\#}$	012345678T	10-2*	<898884>
D	A	$C^{\#}$	E	F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	012346789T	10-6*	<888885>
A	$C^{\#}$	E	F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	012345678T	10-2*	<898884>
$C^{\#}$	E	F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	012345789T	10-5*	<888894>
E	F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	012345689T	10-4*	<888984>
F	B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	012345679T	10-3*	<889884>
B	$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	0123456789	10-1*	<988884>
$A^{\#}$	G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	E	012346789T	10-6*	<888885>
G	$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	E	F	0123456789	10-1*	<988884>
$D^{\#}$	$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	E	F	B	012345679T	10-3*	<889884>
$G^{\#}$	$F^{\#}$	C	D	A	$C^{\#}$	E	F	B	$A^{\#}$	012345689T	10-4*	<888984>
$F^{\#}$	C	D	A	$C^{\#}$	E	F	B	$A^{\#}$	G	012345789T	10-5*	<888894>
C	D	A	$C^{\#}$	E	F	B	$A^{\#}$	G	$D^{\#}$	012345678T	10-2*	<898884>

List of Pitch-class Sets
SI AITN Chord 58
Secondary Resources (alternate for AITN 1 materials)
Literal Spelling: C – D – D# – G – E – B – F – Bb – Db – A – G# – F# – C
21497 6 538ET (6-14)

SI AITN 58 – Trichords:

C	D	D#	013	3-2	<111000>	Phrygian Trichord (023)
D	D#	G	015	3-4	<100110>	Incomplete Major Seventh Chord (045)
D#	G	E	014	3-3	<101100>	Major-minor Trichord 1 (034)
G	E	B	037	3-11	<001110>	Major/Minor Chord (047)
E	B	F	016	3-5	<100011>	Rite Chord/Tritone Fourth (056)
B	F	Bb	016	3-5	<100011>	Rite Chord/Tritone Fourth (056)
F	Bb	Db	037	3-11	<001110>	Major/Minor Chord (047)
Bb	Db	A	014	3-3	<101100>	Major-minor Trichord 1 (034)
Db	A	G#	015	3-4	<100110>	Incomplete Major Seventh Chord (045)
A	G#	F#	013	3-2	<111000>	Phrygian Trichord (023)
G#	F#	C	026	3-8	<010101>	Incomplete Dominant Seventh/Italian Sixth (046)
F#	C	D	026	3-8	<010101>	Incomplete Dominant Seventh/Italian Sixth (046)
C	D	D#	013	3-2	<111000>	Phrygian Trichord (023)

SI AITN 58 – Tetrachords:

C	D	D#	G	0237	4-14<	<111120>	Major-second Minor Tetrachord (0457)
D	D#	G	E	0125	4-4	<211110>	Minor Third Tetracluster 2 (0345)
D#	G	E	B	0148	4-19	<101310>	Minor-augmented Tetrachord (0348)
G	E	B	F	0137	4-z29	<111111>	All-Interval Tetrachord (AIT) 3 (0467)
E	B	F	Bb	0167	4-9*[6]	<200022>	Double Tritone – Mode IV Complement
B	F	Bb	Db	0137	4-z29	<111111>	All-Interval Tetrachord (AIT) 3 (0467)
F	Bb	Db	A	0148	4-19	<101310>	Minor-augmented Tetrachord (0348)
Bb	Db	A	G#	0125	4-4	<211110>	Minor Third Tetracluster 2 (0345)
Db	A	G#	F#	0237	4-14<	<111120>	Major-second Minor Tetrachord (0457)
A	G#	F#	C	0236	4-12<	<112101>	Harmonic-minor Tetrachord (0346)
G#	F#	C	D	0268	4-25*[6]	<020202>	French Sixth – Mode VI Complement
F#	C	D	D#	0236	4-12<	<112101>	Harmonic-minor Tetrachord (0346)
C	D	D#	G	0237	4-14<	<111120>	Major-second Minor Tetrachord (0457)

SI AITN 58 – Pentachords:

C	D	D#	G	E	02347	5-11	<222220>	Centre-cluster Pentachord 1 (03457)
D	D#	G	E	B	03458	5-z37*	<212320>	Centre-cluster Pentamirror
D#	G	E	B	F	01248	5-13	<221311>	Augmented Pentacluster 1 (02348)
G	E	B	F	Bb	01367	5-19	<212122>	Javanese Pentachord (01467)
E	B	F	Bb	Db	01367	5-19	<212122>	Javanese Pentachord (01467)
B	F	Bb	Db	A	01248	5-13	<221311>	Augmented Pentacluster 1 (02348)
F	Bb	Db	A	G#	03458	5-z37*	<212320>	Centre-cluster Pentamirror
Bb	Db	A	G#	F#	02347	5-11	<222220>	Centre-cluster Pentachord 1 (03457)
Db	A	G#	F#	C	01457	5-z18	<212221>	Gypsy Pentachord 1 (02367)
A	G#	F#	C	D	02368	5-28<	<122212>	Augmented-sixth Pentachord1 02568
G#	F#	C	D	D#	02368	5-28<	<122212>	Augmented-sixth Pentachord1 02568
F#	C	D	D#	G	01457	5-z18	<212221>	Gypsy Pentachord 1 (02367)
C	D	D#	G	E	02347	5-11	<222220>	Centre-cluster Pentachord 1 (03457)

SI AITN 58 – Hexachords:

C	D	D#	G	E	B	013458	6-14	<323430>	Combinatorial (034578)
D	D#	G	E	B	F	023458	6-z39	<333321>	6-z39B: 034568
D#	G	E	B	F	Bb	012478	6-z17	<322332>	All-Triad Hexachord (014678)
G	E	B	F	Bb	Db	013679	6-30 [6]	<224223>	Minor-bitonal comb. 023689
E	B	F	Bb	Db	A	012478	6-z17	<322332>	All-Triad Hexachord (014678)
B	F	Bb	Db	A	G#	023458	6-z39	<333321>	6-z39B: 034568
F	Bb	Db	A	G#	F#	013458	6-14	<323430>	Combinatorial (034578)
Bb	Db	A	G#	F#	C	013457	6-z10	<333321>	6-z10B: 023467
Db	A	G#	F#	C	D	012568	6-z43	<322332>	6-z43B: 023678
A	G#	F#	C	D	D#	013679	6-30 [6]	<224223>	Minor-bitonal comb. 023689
G#	F#	C	D	D#	G	012568	6-z43	<322332>	6-z43B: 023678
F#	C	D	D#	G	E	013457	6-z10	<333321>	6-z10B: 023467
C	D	D#	G	E	B	013458	6-14	<323430>	Combinatorial (034578)

SI AITN 58 – Heptachords:

C	D	D#	G	E	B	F	0134568	7-11	<444441>	7-11B: 0234578
D	D#	G	E	B	F	Bb	0234589	7-z18	<434442>	B: 0123589/0146789
D#	G	E	B	F	Bb	Db	0135679	7-28<	<344433>	7-28B: 0234689
G	E	B	F	Bb	Db	A	0135679	7-28<	<344433>	7-28B: 0234689
E	B	F	Bb	Db	A	G#	0234589	7-z18	<434442>	B: 0123589/0146789
B	F	Bb	Db	A	G#	F#	0134568	7-11	<444441>	7-11B: 0234578
F	Bb	Db	A	G#	F#	C	0134578	7-z37*	<434541>	
Bb	Db	A	G#	F#	C	D	0124568	7-13	<443532>	7-13B: 0234678
Db	A	G#	F#	C	D	D#	0123679	7-19	<434343>	7-19B: 0123689
A	G#	F#	C	D	D#	G	0123679	7-19	<434343>	7-19B: 0123689
G#	F#	C	D	D#	G	E	0124568	7-13	<443532>	7-13B: 0234678
F#	C	D	D#	G	E	B	0134578	7-z37*	<434541>	
C	D	D#	G	E	B	F	0134568	7-11	<444441>	7-11B: 0234578

SI AITN 58 – Octachords:

C	D	D#	G	E	B	F	Bb	01245679	8-14<	<555562>	8-14B: 02345789
D	D#	G	E	B	F	Bb	Db	01345679	8-12<	<556543>	8-12B: 02345689
D#	G	E	B	F	Bb	Db	A	0124678T	8-25*	<464644>	Mode VI
G	E	B	F	Bb	Db	A	G#	01345679	8-12<	<556543>	8-12B: 02345689
E	B	F	Bb	Db	A	G#	F#	01245679	8-14<	<555562>	8-14B: 02345789
B	F	Bb	Db	A	G#	F#	C	01234578	8-4	<655552>	8-4B: 01345678
F	Bb	Db	A	G#	F#	C	D	01245689	8-19	<545752>	8-19B: 01345789
Bb	Db	A	G#	F#	C	D	D#	01235679	8-z29	<555553>	8-z29B: 02346789
Db	A	G#	F#	C	D	D#	G	01236789	8-9	<644464>	Mode IV
A	G#	F#	C	D	D#	G	E	01235679	8-z29	<555553>	8-z29B: 02346789
G#	F#	C	D	D#	G	E	B	01245689	8-19	<545752>	8-19B: 01345789
F#	C	D	D#	G	E	B	F	01234578	8-4	<655552>	8-4B: 01345678
C	D	D#	G	E	B	F	Bb	01245679	8-14<	<555562>	8-14B: 02345789

SI AITN 58 – Nonachords:

C	D	D#	G	E	B	F	Bb	Db	012345679	9-2	<777663>	B: 023456789
D	D#	G	E	B	F	Bb	Db	A	01234678T	9-8	<676764>	B: 01234689T
D#	G	E	B	F	Bb	Db	A	G#	01234678T	9-8	<676764>	B: 01234689T
G	E	B	F	Bb	Db	A	G#	F#	012345679	9-2	<777663>	B: 023456789
E	B	F	Bb	Db	A	G#	F#	C	012345789	9-4	<766773>	B: 012456789
B	F	Bb	Db	A	G#	F#	C	D	012345689	9-3	<767763>	B: 013456789
F	Bb	Db	A	G#	F#	C	D	D#	01235679T	9-11	<667773>	B: 01235689T
Bb	Db	A	G#	F#	C	D	D#	G	012346789	9-5	<766674>	B: 012356789
Db	A	G#	F#	C	D	D#	G	E	012346789	9-5	<766674>	B: 012356789
A	G#	F#	C	D	D#	G	E	B	01235679T	9-11	<667773>	B: 01235689T
G#	F#	C	D	D#	G	E	B	F	012345689	9-3	<767763>	B: 013456789
F#	C	D	D#	G	E	B	F	Bb	012345789	9-4	<766773>	B: 012456789
C	D	D#	G	E	B	F	Bb	Db	012345679	9-2	<777663>	B: 023456789

SI AITN 58 – Decachords:

C	D	D#	G	E	B	F	Bb	Db	A	012345678T	10-2*	<898884>
D	D#	G	E	B	F	Bb	Db	A	G#	012346789T	10-6*	<888885>
D#	G	E	B	F	Bb	Db	A	G#	F#	012345678T	10-2*	<898884>
G	E	B	F	Bb	Db	A	G#	F#	C	0123456789	10-1*	<988884>
E	B	F	Bb	Db	A	G#	F#	C	D	012345689T	10-4*	<888984>
B	F	Bb	Db	A	G#	F#	C	D	D#	012345679T	10-3*	<889884>
F	Bb	Db	A	G#	F#	C	D	D#	G	012345789T	10-5*	<888894>
Bb	Db	A	G#	F#	C	D	D#	G	E	012346789T	10-6*	<888885>
Db	A	G#	F#	C	D	D#	G	E	B	012345789T	10-5*	<888894>
A	G#	F#	C	D	D#	G	E	B	F	012345679T	10-3*	<889884>
G#	F#	C	D	D#	G	E	B	F	Bb	012345689T	10-4*	<888984>
F#	C	D	D#	G	E	B	F	Bb	Db	0123456789	10-1*	<988884>
C	D	D#	G	E	B	F	Bb	Db	A	012345678T	10-2*	<898884>

List of Pitch-class Sets
SI AITN Chord 60
Primary Resources
Literal Spelling: C – D – E^{b} – G – B^{b} – F – B – E – $C^{\#}$ – A – $G^{\#}$ – $F^{\#}$ – C
21437 6 598ET (6-32)

SI AITN 60 – Trichords:

C	D	E^{b}	013	3-2	<111000>	Phrygian Trichord (023)
D	E^{b}	G	015	3-4	<100110>	Incomplete Major Seventh Chord (045)
E^{b}	G	B^{b}	037	3-11	<001110>	Major/Minor Chord (047)
G	B^{b}	F	025	3-7	<011010>	Incomplete minor seventh Chord (035)
B^{b}	F	B	016	3-5	<100011>	Rite Chord/Tritone Fourth (056)
F	B	E	016	3-5	<100011>	Rite Chord/Tritone Fourth (056)
B	E	$C^{\#}$	025	3-7	<011010>	Incomplete minor seventh Chord (035)
E	$C^{\#}$	A	037	3-11	<001110>	Major/Minor Chord (047)
$C^{\#}$	A	$G^{\#}$	015	3-4	<100110>	Incomplete Major Seventh Chord (045)
A	$G^{\#}$	$F^{\#}$	013	3-2	<111000>	Phrygian Trichord (023)
$G^{\#}$	$F^{\#}$	C	026	3-8	<010101>	Incomplete Dominant Seventh/Italian Sixth (046)
$F^{\#}$	C	D	026	3-8	<010101>	Incomplete Dominant Seventh/Italian Sixth (046)
C	D	E^{b}	013	3-2	<111000>	Phrygian Trichord (023)

SI AITN 60 – Tetrachords:

C	D	E^{b}	G	0237	4-14<	<111120>	Major-second Minor Tetrachord (0457)
D	E^{b}	G	B^{b}	0158	4-20*	<101220>	Major Seventh Chord
E^{b}	G	B^{b}	F	0247	4-22	<021120>	Major-second Major Tetrachord (0357)
G	B^{b}	F	B	0146	4-z15	<111111>	All-Interval Tetrachord 1 (0256)
B^{b}	F	B	E	0167	4-9*[6]	<200022>	Double Tritone – Mode IV Complement
F	B	E	$C^{\#}$	0146	4-z15	<111111>	All-Interval Tetrachord 1 (0256)
B	E	$C^{\#}$	A	0247	4-22	<021120>	Major-second Major Tetrachord (0357)
E	$C^{\#}$	A	$G^{\#}$	0158	4-20*	<101220>	Major Seventh Chord
$C^{\#}$	A	$G^{\#}$	$F^{\#}$	0237	4-14<	<111120>	Major-second Minor Tetrachord (0457)
A	$G^{\#}$	$F^{\#}$	C	0236	4-12<	<112101>	Harmonic-minor Tetrachord (0346)
$G^{\#}$	$F^{\#}$	C	D	0268	4-25*[6]	<020202>	French Sixth – Mode VI Complement
$F^{\#}$	C	D	E^{b}	0236	4-12<	<112101>	Harmonic-minor Tetrachord (0346)
C	D	E^{b}	G	0237	4-14<	<111120>	Major-second Minor Tetrachord (0457)

SI AITN 60 – Pentachords:

C	D	E^{b}	G	B^{b}	01358	5-27	<122230>	Major-Ninth Chord (03578)
D	E^{b}	G	B^{b}	F	01358	5-27	<122230>	Major-Ninth Chord (03578)
E^{b}	G	B^{b}	F	B	01468	5-30	<121321>	Enigmatic Pentachord 1 (02478)
G	B^{b}	F	B	E	01367	5-19	<212122>	Javanese Pentachord (01467)
B^{b}	F	B	E	C$^{\#}$	01367	5-19	<212122>	Javanese Pentachord (01467)
F	B	E	C$^{\#}$	A	01468	5-30	<121321>	Enigmatic Pentachord 1 (02478)
B	E	C$^{\#}$	A	G$^{\#}$	01358	5-27	<122230>	Major-Ninth Chord (03578)
E	C$^{\#}$	A	G$^{\#}$	F$^{\#}$	01358	5-27	<122230>	Major-Ninth Chord (03578)
C$^{\#}$	A	G$^{\#}$	F$^{\#}$	C	01457	5-z18	<212221>	Gypsy Pentachord 1 (02367)
A	G$^{\#}$	F$^{\#}$	C	D	02368	5-28<	<122212>	Augmented-sixth Pentachord1 02568
G$^{\#}$	F$^{\#}$	C	D	E^{b}	02368	5-28<	<122212>	Augmented-sixth Pentachord1 02568
F$^{\#}$	C	D	E^{b}	G	01457	5-z18	<212221>	Gypsy Pentachord 1 (02367)
C	D	E^{b}	G	B^{b}	01358	5-27	<122230>	Major-Ninth Chord (03578)

SI AITN 60 – Hexachords:

C	D	E^{b}	G	B^{b}	F	024579	6-32*	<143250>	Arezzo Major Diatonic Major Quartal hexamirror
D	E^{b}	G	B^{b}	F	B	014579	6-31	<223431>	Combinatorial I (7) (013589)
E^{b}	G	B^{b}	F	B	E	012478	6-z17	<322332>	All-Triad Hexachord (014678)
G	B^{b}	F	B	E	C$^{\#}$	013679	6-30 [6]	<224223>	Minor-bitonal comb. 023689
B^{b}	F	B	E	C$^{\#}$	A	012478	6-z17	<322332>	All-Triad Hexachord (014678)
F	B	E	C$^{\#}$	A	G$^{\#}$	014579	6-31	<223431>	Combinatorial I (7) (013589)
B	E	C$^{\#}$	A	G$^{\#}$	F$^{\#}$	024579	6-32*	<143250>	Arezzo Major Diatonic Major Quartal hexamirror
E	C$^{\#}$	A	G$^{\#}$	F$^{\#}$	C	014579	6-31	<223431>	Combinatorial I (7) (013589)
C$^{\#}$	A	G$^{\#}$	F$^{\#}$	C	D	012568	6-z43	<322332>	6-z43B: 023678
A	G$^{\#}$	F$^{\#}$	C	D	E^{b}	013679	6-30	<224223>	Minor-bitonal comb. 023689
G$^{\#}$	F$^{\#}$	C	D	E^{b}	G	012568	6-z43	<322332>	6-z43B: 023678
F$^{\#}$	C	D	E^{b}	G	B^{b}	014579	6-31	<223431>	Combinatorial I (7) (013589)
C	D	E^{b}	G	B^{b}	F	024579	6-32	<143250>	Arezzo Major Diatonic Major Quartal hexamirror

SI AITN 60 – Heptachords:

C	D	E^{b}	G	B^{b}	F	B	0124579	7-27	<344451>	7-27B: 0245789 Modified Blues Scale
D	E^{b}	G	B^{b}	F	B	E	0234589	7-z18	<434442>	B: 0123589/0146789
E^{b}	G	B^{b}	F	B	E	$C^{\#}$	0135679	7-28<	<344433>	7-28B: 0234689
G	B^{b}	F	B	E	$C^{\#}$	A	0135679	7-28<	<344433>	7-28B: 0234689
B^{b}	F	B	E	$C^{\#}$	A	$G^{\#}$	0234589	7-z18	<434442>	B: 0123589/0146789
F	B	E	$C^{\#}$	A	$G^{\#}$	$F^{\#}$	0124579	7-27	<344451>	7-27B: 0245789
B	E	$C^{\#}$	A	$G^{\#}$	$F^{\#}$	C	0124579	7-27	<344451>	7-27B: 0245789
E	$C^{\#}$	A	$G^{\#}$	$F^{\#}$	C	D	0124689 0135789	7-30	<343542>	Neapolitan-Minor (1222131) Mela Dhenuka
$C^{\#}$	A	$G^{\#}$	$F^{\#}$	C	D	E^{b}	0123679	7-19	<434343>	7-19B: 0123689
A	$G^{\#}$	$F^{\#}$	C	D	E^{b}	G	0123679	7-19	<434343>	7-19B: 0123689
$G^{\#}$	$F^{\#}$	C	D	E^{b}	G	B^{b}	0124689 0135789	7-30	<343542>	Neapolitan-Minor (1222131) Mela Dhenuka
$F^{\#}$	C	D	E^{b}	G	B^{b}	F	0124579	7-27	<344451>	7-27B: 0245789
C	D	E^{b}	G	B^{b}	F	B	0124579	7-27	<344451>	7-27B: 0245789

SI AITN 60 – Octachords:

C	D	E^{b}	G	B^{b}	F	B	E	01245679	8-14<	<555562>	8-14B: 02345789
D	E^{b}	G	B^{b}	F	B	E	$C^{\#}$	01345679	8-12<	<556543>	8-12B: 02345689
E^{b}	G	B^{b}	F	B	E	$C^{\#}$	A	0124678T	8-25*	<464644>	Mode VI
G	B^{b}	F	B	E	$C^{\#}$	A	$G^{\#}$	01345679	8-12<	<556543>	8-12B: 02345689
B^{b}	F	B	E	$C^{\#}$	A	$G^{\#}$	$F^{\#}$	01245679	8-14<	<555562>	8-14B: 02345789
F	B	E	$C^{\#}$	A	$G^{\#}$	$F^{\#}$	C	01245789	8-20*	<545662>	
B	E	$C^{\#}$	A	$G^{\#}$	$F^{\#}$	C	D	0123568T	8-22	<465562>	8-22B: 0123579T Spanish Octatonic
E	$C^{\#}$	A	$G^{\#}$	$F^{\#}$	C	D	E^{b}	01234689	8-z15	<555553>	8-z15B: 01356789
$C^{\#}$	A	$G^{\#}$	$F^{\#}$	C	D	E^{b}	G	01236789	8-9	<644464>	Mode IV
A	$G^{\#}$	$F^{\#}$	C	D	E^{b}	G	B^{b}	01234689	8-z15	<555553>	8-z15B: 01356789
$G^{\#}$	$F^{\#}$	C	D	E^{b}	G	B^{b}	F	0123568T	8-22	<465562>	8-22B: 0123579T Spanish Octatonic
$F^{\#}$	C	D	E^{b}	G	B^{b}	F	B	01245789	8-20*	<545662>	
C	D	E^{b}	G	B^{b}	F	B	E	01245679	8-14<	<555562>	8-14B: 02345789

SI AITN 60 – Nonachords:

C	D	E^b	G	B^b	F	B	E	$C^\#$	012345679	9-2	<777663>	B: 023456789
D	E^b	G	B^b	F	B	E	$C^\#$	A	01234678T	9-8	<676764>	B: 01234689T
E^b	G	B^b	F	B	E	$C^\#$	A	$G^\#$	01234678T	9-8	<676764>	B: 01234689T
G	B^b	F	B	E	$C^\#$	A	$G^\#$	$F^\#$	012345679	9-2	<777663>	B: 023456789
B^b	F	B	E	$C^\#$	A	$G^\#$	$F^\#$	C	012345789	9-4	<766773>	B: 012456789
F	B	E	$C^\#$	A	$G^\#$	$F^\#$	C	D	01235679T	9-11	<667773>	B: 01235689T
B	E	$C^\#$	A	$G^\#$	$F^\#$	C	D	E^b	01234578T	9-7	<677673>	B: 01234579T NonatonicBlues
E	$C^\#$	A	$G^\#$	$F^\#$	C	D	E^b	G	012346789	9-5	<766674>	B: 012356789
$C^\#$	A	$G^\#$	$F^\#$	C	D	E^b	G	B^b	012346789	9-5	<766674>	B: 012356789
A	$G^\#$	$F^\#$	C	D	E^b	G	B^b	F	01234578T	9-7	<677673>	B: 01234579T NonatonicBlues
$G^\#$	$F^\#$	C	D	E^b	G	B^b	F	B	01235679T	9-11	<667773>	B: 01235689T
$F^\#$	C	D	E^b	G	B^b	F	B	E	012345789	9-4	<766773>	B: 012456789
C	D	E^b	G	B^b	F	B	E	$C^\#$	012345679	9-2	<777663>	B: 023456789

SI AITN 60 – Decachords:

C	D	E^b	G	B^b	F	B	E	$C^\#$	A	012345678T	10-2*	<898884>
D	E^b	G	B^b	F	B	E	$C^\#$	A	$G^\#$	012346789T	10-6*	<888885>
E^b	G	B^b	F	B	E	$C^\#$	A	$G^\#$	$F^\#$	012345678T	10-2*	<898884>
G	B^b	F	B	E	$C^\#$	A	$G^\#$	$F^\#$	C	0123456789	10-1*	<988884>
B^b	F	B	E	$C^\#$	A	$G^\#$	$F^\#$	C	D	012345689T	10-4*	<888984>
F	B	E	$C^\#$	A	$G^\#$	$F^\#$	C	D	E^b	012345679T	10-3*	<889884>
B	E	$C^\#$	A	$G^\#$	$F^\#$	C	D	E^b	G	012345789T	10-5*	<888894>
E	$C^\#$	A	$G^\#$	$F^\#$	C	D	E^b	G	B^b	012346789T	10-6*	<888885>
$C^\#$	A	$G^\#$	$F^\#$	C	D	E^b	G	B^b	F	012345789T	10-5*	<888894>
A	$G^\#$	$F^\#$	C	D	E^b	G	B^b	F	B	012345679T	10-3*	<889884>
$G^\#$	$F^\#$	C	D	E^b	G	B^b	F	B	E	012345689T	10-4*	<888984>
$F^\#$	C	D	E^b	G	B^b	F	B	E	$C^\#$	0123456789	10-1*	<988884>
C	D	E^b	G	B^b	F	B	E	$C^\#$	A	012345678T	10-2*	<898884>