

Rotations and Resonances

For amplified string quartet, interactive sound, concrete tape, amplified piano resonances and live electronics

Heterodyning Relationships for A^b

Difference tones for A^b2 [103.83]: C4 equals middle C, A4 = 440

$$B^b6 [1864.7] \text{ minus } A6 [1760] = 104.7$$

$$B^b5 [932.33] \text{ minus } A^b5 [830.61] = 101.72$$

$$E5 [659.26] \text{ minus } D^b5 [554.37] = 104.89$$

$$B4 [493.88] \text{ minus } G4 [392] = 101.88$$

$$A^b4 [415.30] \text{ minus } E^b4 [311.13] = 104.17 \text{ *E}^b4 \text{ appears twice*}$$

$$F4 [349.23] \text{ minus } B3 [246.94] = 102.27$$

$$E^b4 [311.13] \text{ minus } A^b3 [207.65] = 103.48 \text{ *E}^b4 \text{ appears twice*}$$

$$D^b4 [277.18] \text{ minus } F3 [174.61] = 102.57$$

$$C4 [261.6] \text{ minus } E^b3 [155.56] = 106.04 \text{ (close to mid-point, but still favouring } A^b2)$$

$$B^b3 [233.08] \text{ minus } C3 [130.81] = 102.27$$

$$A3 [220] \text{ minus } B^b2 [116.54] = 103.46$$

$$G3 [196] \text{ minus } F\#2 [92.499] = 103.501$$

$$F\#3 [185] \text{ minus } E2 [82.407] = 102.593$$

$$F3 [174.61] \text{ minus } D^b2 [69.296] = 105.314$$

Note: The lowest frequency available in the string quartet is C2 [65.406]

B^b6 & A6 (semi-tone)

B^b5 & A^b5 (tone or major second)

E5 & D^b5 (minor-third)

B4 & G4 (major third)

A^b4 & E^b4 (perfect fourth) *E^b4 appears twice*

F4 & B3 (tritone)

E^b4 & A^b3 (perfect fifth) *E^b4 appears twice*

D^b4 & F3 (augmented fifth or minor sixth)

C4 & E^b3 (major sixth)

B^b3 & C3 (flattened seventh)

A3 & B^b2 (major seventh)

G3 & F[#]2 (semi-tone)

F[#]3 & E2 (tone or major second)

F3 & D^b2 (major third)

Note:

Total of 14 difference tones playable by a string quartet for A^b2, with one pair omitted because although their closest concert pitch frequency is A^b2, it is also very close to halfway between the two 'concert pitch' fundamentals:

F3 [174.61] minus D2 [73.416] = 101.194 (between fundamentals of A^b2 and G2)

F3 & D2 (compound minor third or sharp ninth) *F3 appears twice if this is included*

Interestingly enough, once one of the pitches falls below the fundamental frequency, a compound interval is produced and the lower the two pitches get (and therefore closer to the fundamental pitch), the wider the interval gets (ignoring some octave transfer).

Combined tones for A^b2 [103.83]: C4 equals middle C, A4 = 440

Note:

The lowest frequency available in the string quartet is C2 [65.406] so no combined tones can be played for A^b2 in this piece.

Difference tones for A^b7 [3322.4]: C4 equals middle C, A4 = 440

C8 [4186] minus A5 [880] = 3306 (OK)
B7 [3951.1] minus E^b5 [622.25] = 3328.85
B^b7 [3729.3] minus A^b4 [415.3] = 3314
A7 [3520] minus G3 [196] = 3324

C8 & A5 (minor third)
B7 & E^b5 (augmented fifth or minor sixth)
B^b7 & A^b4 (tone or major second)
A7 & G3 (tone or major second)

Combined tones for A^b7 [3322.4]: C4 equals middle C, A4 = 440

F[#]3 [185] plus G7 [3136] = 3321
F[#]4 [369.99] plus F[#]7 [2960] = 3329.99
C5 [523.25] plus F7 [2793.8] = 3317.05
F5 [698.46] plus E7 [2637] = 3335.46
A^b5 [830.61] plus E^b7 [2489] = 3319.61
B5 [987.77] plus D7 [2349.3] = 3337.07
D^b6 [1108.7] plus D^b7 [2217.5] = 3326.2
E^b6 [1244.5] plus C7 [2093] = 3337.5
E6 [1318.5] plus B6 [1975.5] = 3294 (A^b7 is closest concert pitch, but most distant)
F[#]6 [1480] plus B^b6 [1864.7] = 3344.7
G6 [1568] plus A6 [1760] = 3328

There are 11 combined tones for A^b7 playable by a string quartet:

F[#]3 & G7 (semi-tone)
F[#]4 & F[#]7 (unison/octave)
C5 & F7 (perfect fourth)
F5 & E7 (major seventh)
A^b5 & E^b7 (perfect fifth)
B5 & D7 (minor third)
D^b6 & D^b7 (unison/octave)
E^b6 & C7 (major sixth)
E6 & B6 (perfect fifth)
F[#]6 & B^b6 (major third)
G6 & A6 (tone or major second)

Note: A^b7 seems quite high for an important harmonic note, so one would prefer to use the frequencies related to A^b6 (below). This list is transposable for use with A^b6.

Combined tones for A^b6 [1661.2]: C4 equals middle C, A4 = 440

F[#]2 [92.499] plus G6 [1568] = 1660.499 (closest)
G2 [97.999] plus G6 [1568] = 1665.99
F3 [174.61] plus F[#]6 [1480] = 1654.61
F[#]3 [185] plus F[#]6 [1480] = 1665 (closest)
C4 [261.6] plus F6 [1396.9] = 1658.5 (closest)
D^b4 [277.18] plus F6 [1396.9] = 1674.08
E4 [329.63] plus E6 [1318.5] = 1648.13
F4 [349.23] plus E6 [1318.5] = 1667.73 (closest)
A^b4 [415.30] plus E^b6 [1244.5] = 1659.8 (closest)
A4 [440] plus E^b6 [1244.5] = 1684.5
B^b4 [466.16] plus D6 [1174.7] = 1640.86
B4 [493.88] plus D6 [1174.7] = 1668.58 (closest)
C5 [523.25] plus D^b6 [1108.7] = 1631.95
D^b5 [554.37] plus D^b6 [1108.7] = 1663.07 (closest)
D5 [587.33] plus C6 [1046.5] = 1633.83
E^b5 [622.25] plus C6 [1046.5] = 1668.75 (closest)
E5 [659.26] plus B5 [987.77] = 1647.03 (closest)
F5 [698.46] plus B5 [987.77] = 1686.23
F5 [698.46] plus B^b5 [932.33] = 1630.79
F[#]5 [739.99] plus B^b5 [932.33] = 1672.32 (closest)
G5 [783.99] plus A5 [880] = 1663.99

Below are the 11 (closest) combined tones for A^b6 playable by the string quartet:

F[#]2 [92.499] plus G6 [1568] = 1660.499
F[#]3 [185] plus F[#]6 [1480] = 1665
C4 [261.6] plus F6 [1396.9] = 1658.5
F4 [349.23] plus E6 [1318.5] = 1667.73
A^b4 [415.30] plus E^b6 [1244.5] = 1659.8
B4 [493.88] plus D6 [1174.7] = 1668.58
D^b5 [554.37] plus D^b6 [1108.7] = 1663.07
E^b5 [622.25] plus C6 [1046.5] = 1668.75
E5 [659.26] plus B5 [987.77] = 1647.03
F[#]5 [739.99] plus B^b5 [932.33] = 1672.32
G5 [783.99] plus A5 [880] = 1663.99

F[#]2 & G6 (semi-tone)
F[#]3 & F[#]6 (octave/unison)
C4 & F6 (perfect fourth)
F4 & E6 (major seventh)
A^b4 & E^b6 (perfect fifth)
B4 & D6 (minor third)

D^b5 & D^b6 (octave/unison)
E^b5 & C6 (major sixth)
E5 & B5 (perfect fifth)
F[#]5 & B^b5 (major third)
G5 & A5 (tone or major second)

Note:

A^b6 could be used as an opposing/complimentary line placed against/with the frequencies for A^b2 accompanying the introduction and featured after the crescendo (before the acoustic epilogue).

Difference tones for A^b6 [1661.2]: C4 equals middle C, A4 = 440

C8 [4186] minus E^b7 [2489] = 1697
B^b7 [3729.3] minus C7 [2093] = 1636.3
A7 [3520] minus B^b6 [1864.7] = 1655.3
G7 [3136] minus F[#]6 [1480] = 1656
F[#]7 [2960] minus E6 [1318.5] = 1641.5
F7 [2793.8] minus D^b6 [1108.7] = 1685.1
E7 [2637] minus B5 [987.77] = 1649.23
E^b7 [2489] minus A^b5 [830.61] = 1658.39
D7 [2349.3] minus F5 [698.46] = 1650.84
D^b7 [2217.5] minus D^b5 [554.37] = 1663.13
C7 [2093] minus A4 [440] = 1653
B6 [1975.5] minus E^b4 [311.13] = 1664.37
B^b6 [1864.7] minus A^b3 [207.65] = 1657.05
A6 [1760] minus G2 [97.999] = 1662.001

There are fourteen approximate difference tones for A^b6:

C8 & E ^b 7 (major sixth)	E ^b 7 & A ^b 5 (perfect fifth)
B ^b 7 & C7 (flat seventh)	D7 & F5 (major sixth)
A7 & B ^b 6 (major seventh)	D ^b 7 & D ^b 5 (unison/octave)
G7 & F [#] 6 (semi-tone)	C7 & A4 (minor third)
F [#] 7 & E6 (major second)	B6 & E ^b 4 (augmented 5 th or minor 6 th)
F7 & D ^b 6 (major third)	B ^b 6 & A ^b 3 (tone or major second)
E7 & B5 (perfect fourth)	A6 & G2 (tone or major second)

Note:

The lowest frequency available in the string quartet is C2 [65.406] so no combined tones can be played for A^b2 in this piece. Consider only employing combined tones for A^b6 and only using difference tones for A^b2, as this would offer a sense of symmetry and completion between the two lines of harmonic texture.

Note:

These lists are transposable to all octaves/registers of A^b and the calculations for all A^b registers have been checked, with the lists given below.

All Calculated Difference Tones for A^b

(Numbered from 1-22)

Difference tones for A^b7 [3322.4]: C4 equals middle C, A4 = 440

19. C8 & A5 (minor third)
20. B7 & E^b5 (augmented fifth or minor sixth)
21. B^b7 & A^b4 (tone or major second)
22. A7 & G3 (tone or major second)

Note:

The next step would be from A^b7 (the fundamental) producing an equal division. All A^b octaves fit with heterodyning theory so this is not considered within my lists. No difference tones exist if the upper note is lower than the fundamental.

Difference tones for A^b6 [1661.2]: C4 equals middle C, A4 = 440

9. C8 & E^b7 (major sixth)
10. B^b7 & C7 (flat seventh)
11. A7 & B^b6 (major seventh)
12. G7 & F[#]6 (semi-tone)
13. F[#]7 & E6 (tone or major second)
14. F7 & D^b6 (major third)
15. E7 & B5 (perfect fourth)
16. E^b7 & A^b5 (perfect fifth)
17. D7 & F5 (major sixth)
18. D^b7 & D^b5 (unison/octave)
19. C7 & A4 (minor third)
20. B6 & E^b4 (augmented fifth or minor sixth)
21. B^b6 & A^b3 (tone or major second)
22. A6 & G2 (tone or major second)

Note:

No difference tones exist if the upper note is lower than the fundamental, so therefore, the next step on this list would be F[#]6 & G7 (notice reverse order) and would descend and rotate through the list from steps 4-14 continuously. This obviously produces a perpetual cycle.

Difference tones for A^b5 [830.61]: C4 equals middle C, A4 = 440

4. B7 & G7 (major third)
5. A^b7 & E^b7 (perfect fourth)
6. F7 & B6 (tritone)
7. E^b7 & A^b6 (perfect fifth)
8. D^b7 & F6 (augmented fifth or minor sixth)
9. C7 & E^b6 (major sixth)
10. B^b6 & C6 (flat seventh)
11. A6 & B^b5 (major seventh)
12. G6 & F[#]5 (semi-tone)
13. F[#]6 & E5 (tone or major second)
14. F6 & D^b5 (major third)
15. E6 & B4 (perfect fourth)
16. E^b6 & A^b4 (perfect fifth)
17. D6 & F4 (major sixth)
18. D^b6 & D^b4 (unison/octave)
19. C6 & A3 (minor third)
20. B5 & E^b3 (augmented fifth or minor sixth)
21. B^b5 & A^b2 (tone or major second)
22. A5 & G1 (tone or major second) **Note:** Not playable by string quartet

Difference tones for A^b4 [415.30]: C4 equals middle C, A4 = 440

1. B^b8 & A8 (semi-tone) **Note:** Not playable
2. B^b7 & A^b7 (tone or major second)
3. E7 & D^b7 (minor third)
4. B6 & G6 (major third)
5. A^b6 & E^b6 (perfect fourth)
6. F6 & B5 (tritone)
7. E^b6 & A^b5 (perfect fifth)
8. D^b6 & F5 (augmented fifth or minor sixth)
9. C6 & E^b5 (major sixth)
10. B^b5 & C5 (flat seventh)
11. A5 & B^b4 (major seventh)
12. G5 & F[#]4 (semi-tone)
13. F[#]5 & E4 (tone or major second)
14. F5 & D^b4 (major third)
15. E5 & B3 (perfect fourth)
16. E^b5 & A^b3 (perfect fifth)
17. D5 & F3 (major sixth)
18. D^b5 & D^b3 (unison/octave)
19. C5 & A2 (minor third)
20. B4 & E^b2 (augmented fifth or minor sixth)
21. B^b4 & A^b1 (tone or major second) **Note:** Not playable by string quartet

Difference tones for A^b3 [207.65]: C4 equals middle C, A4 = 440

1. B^b7 & A7 (semi-tone)
2. B^b6 & A^b6 (tone or major second)
3. E6 & D^b6 (minor third)
4. B5 & G5 (major third)
5. A^b5 & E^b5 (perfect fourth)
6. F5 & B4 (tritone)
7. E^b5 & A^b4 (perfect fifth)
8. D^b5 & F4 (augmented fifth or minor sixth)
9. C5 & E^b4 (major sixth)
10. B^b4 & C4 (flat seventh)
11. A4 & B^b3 (major seventh)
12. G4 & F[#]3 (semi-tone)
13. F[#]4 & E3 (tone or major second)
14. F4 & D^b3 (major third)
15. E4 & B2 (perfect fourth)
16. E^b4 & A^b2 (perfect fifth)
17. D4 & F2 (major sixth)
18. D^b4 & D^b2 (unison/octave)
19. C4 & A1 (minor third) **Note:** Not playable by string quartet
20. B3 & E^b1 (augmented fifth or minor sixth)
21. B^b3 & A^b0 (tone or major second) **Note:** Not playable

Difference tones for A^b2 [103.83]: C4 equals middle C, A4 = 440

1. B^b6 & A6 (semi-tone)
2. B^b5 & A^b5 (tone or major second)
3. E5 & D^b5 (minor third)
4. B4 & G4 (major third)
5. A^b4 & E^b4 (perfect fourth)
6. F4 & B3 (tritone)
7. E^b4 & A^b3 (perfect fifth)
8. D^b4 & F3 (augmented fifth or minor sixth)
9. C4 & E^b3 (major sixth)
10. B^b3 & C3 (flat seventh)
11. A3 & B^b2 (major seventh)
12. G3 & F[#]2 (semi tone)
13. F[#]3 & E2 (tone or major second)
14. F3 & D^b2 (major third)
15. E3 & B1 (perfect fourth) **Note:** Not playable by string quartet
16. E^b3 & A^b1 (perfect fifth)
17. D3 & F1 (major sixth)
18. D^b3 & D^b1 (unison/octave)
19. C3 & A0 (minor third)

Difference tones for A^b1 [51.913]: C4 equals middle C, A4 = 440

1. B^b5 & A5 (semi-tone)
2. B^b4 & A^b4 (tone or major second)
3. E4 & D^b4 (minor third)
4. B3 & G3 (major third)
5. A^b3 & E^b3 (perfect fourth)
6. F3 & B2 (tritone)
7. E^b3 & A^b2 (perfect fifth)
8. D^b3 & F2 (augmented fifth or minor sixth)
9. C3 & E^b2 (major sixth)
10. B^b2 & C2 (flat seventh)
11. A2 & B^b1 (major seventh) **Note:** Not playable by string quartet
12. G2 & F[#]1 (semi tone)
13. F[#]2 & E1 (tone or major second)
14. F2 & D^b1 (major third)
15. E2 & B0 (perfect fourth)
16. E^b2 & A^b0 (perfect fifth) **Note:** Not playable
17. D2 & F0 (major sixth)
18. D^b2 & D^b0
19. C2 & A***
20. B1 & E^b***
21. B^b1 & A^b***
22. A1 & G***

All Calculated Combined Tones for A^b

Combined tones for A^{b7} [3322.4]: C4 equals middle C, A4 = 440

1. G7 & F[#]3 (semi-tone)
2. F[#]7 & F[#]4 (unison/octave)
3. F7 & C5 (perfect fourth)
4. E7 & F5 (major seventh)
5. E^b7 & A^b5 (perfect fifth)
6. D7 & B5 (minor third)
7. D^b7 & D^b6 (unison/octave)
8. C7 & E^b6 (major sixth)
9. B6 & E6 (perfect fifth)
10. B^b6 & F[#]6 (major third)
11. A6 & G6 (tone or major second)

Note:

The next ascending step would be A^{b7} (the fundamental). No 'concert pitch' combined tones can exist once the highest pitch is above that of the fundamental. The next descending step would be A^{b6} & A^{b6}. All octave pairs fit with heterodyning theory so this is not considered within these lists.

Once the frequencies of both tones falls below half that of the fundamental (octave below), they can no longer combine to equate the fundamental frequency. In order to extend the above list one would then have to use three-part harmony.

A simple form of three-part combined-tone harmony for A^{b7} without researching further than these listed calculations would be to use an A^{b6} [1661.2] accounting for half the fundamental frequency and set it alongside a pair of combined tones from A^{b6} (listed below). However, this would only produce octave doubles and unisons of the same list when far more variants can be considered.

Note:

Combined tones move in contrary motion towards the central pitch an octave below the fundamental (i.e. combined tones for A^{b7} head towards A^{b6}). Unlike difference tones, which rotate when they reach this scenario, combined tones revolve: 1 to 11, 11 to 1 etc. to produce a perpetual cycle. Number 12 on the above list would actually be G6 & A6 (note reverse order).

Combined tones for A^b6 [1661.2]: C4 equals middle C, A4 = 440

1. G⁶ & F[#]2 (semi-tone)
2. F[#]6 & F[#]3 (unison/octave)
3. F6 & C4 (perfect fourth)
4. E6 & F4 (major seventh)
5. E^b6 & A^b4 (perfect fifth)
6. D6 & B4 (minor third)
7. D^b6 & D^b5 (unison/octave)
8. C6 & E^b5 (major sixth)
9. B5 & E5 (perfect fifth)
10. B^b5 & F[#]5 (major third)
11. A5 & G5 (tone or major second)

Combined tones for A^b5 [830.61]: C4 equals middle C, A4 = 440

1. G5 & F[#]1 (semi-tone) **Note:** Not playable by string quartet:
2. F[#]5 & F[#]2 (unison/octave)
3. F5 & C3 (perfect fourth)
4. E5 & F3 (major seventh)
5. E^b5 & A^b3 (perfect fifth)
6. D5 & B3 (minor third)
7. D^b5 & D^b4 (unison/octave)
8. C5 & E^b4 (major sixth)
9. B4 & E4 (perfect fifth)
10. B^b4 & F[#]4 (major third)
11. A4 & G4 (tone or major second)

Combined tones for A^b4 [415.30]: C4 equals middle C, A4 = 440

1. G4 & F[#]0 (semi-tone) **Note:** Not playable
2. F[#]4 & F[#]1 (unison/octave) **Note:** Not playable by string quartet
3. F4 & C2 (perfect fourth)
4. E4 & F2 (major seventh)
5. E^b4 & A^b2 (perfect fifth)
6. D4 & B2 (minor third)
7. D^b4 & D^b3 (unison/octave)
8. C4 & E^b3 (major sixth)
9. B3 & E3 (perfect fifth)
10. B^b3 & F[#]3 (major third)
11. A3 & G3 (tone or major second)

Combined tones for A^b3 [207.65]: C4 equals middle C, A4 = 440

1. G³ & F[#]*** **Note:** Not playable
2. F[#]3 & F[#]0 (unison/octave) **Note:** Not playable
3. F3 & C1 (perfect fourth) **Note:** Not playable by string quartet
4. E3 & F1 (major seventh) **Note:** Not playable by string quartet
5. E^b3 & A^b1 (perfect fifth) **Note:** Not playable by string quartet
6. D3 & B1 (minor third) **Note:** Not playable by string quartet
7. D^b3 & D^b2 (unison/octave)
8. C3 & E^b2 (major sixth)
9. B2 & E2 (perfect fifth)
10. B^b2 & F[#]2 (major third)
11. A2 & G2 (tone or major second)

Combined tones for A^b2 [103.83]: C4 equals middle C, A4 = 440

1. G2 & F[#]***
2. F[#]2 & F[#]***
3. F2 & C0 (perfect fourth) **Note:** Not playable
4. E2 & F0 (major seventh) **Note:** Not playable
5. E^b2 & A^b0 (perfect fifth) **Note:** Not playable
6. D2 & B0 (minor third) **Note:** Not playable by string quartet:
7. D^b2 & D^b1 (unison/octave)
8. C2 & E^b1 (major sixth)
9. B1 & E1 (perfect fifth)
10. B^b1 & F[#]1 (major third)
11. A1 & G1 (tone or major second)

Note:

The lowest frequency available in the string quartet is C2 [65.406] so no combined tones can be played for A^b2 in this piece.