

Pitch Organisation in the 21st Century

Pitch Palindromes from the Micro to the Macro

[What goes up must come down]

This document summarises a period of theoretical and practical research on the subject of pitch palindromes¹. This line of research has influenced many recent works and has provided the foundations for a current orchestral 'work in progress' but is extensively featured in the scores and theory for *The Palindrome Triptychs*.

Pitch Palindromes, and the potential for exploring palindromes within palindromes, adds another musical stratum within an evolving concept of Matryoshka Form² and self-similar replication which emerged from recent consideration of pitch organisation and proportional subcomponents of internal architecture relative to the study of panoptic non-linear resonances, fractals and chaos theory.

The potential for the connectives outlined in this document seem limitless (one has only uncovered the tip of an iceberg here). This document simply lists observations about the inherent relationships between particular intervals and their vast universe of combinations. What style one chooses to embrace is not predetermined by the pitch organisation outlined here.

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¹ A word or phrase, the letters (or components) of which, when taken in reverse order read the same

² A design paradigm indicating the existence of object within similar object, originating from the Russian 'nested' dolls design. It is important to note that Matryoshka dolls are traditionally hand painted. They are not exact replications, but self-similar replications. Their subtle differences make each unique ...

Pitch Palindromes from the Micro to the Macro

(What goes up must come down)

1. Unison (Octave) – The details of the individual note:
 - i. The unison is a static palindrome frozen in time (vertical time).
 - ii. An isolated pitch is therefore (theoretically) the smallest unit through which to present a symmetrical, self-contained, vertical and linear, pitch and rhythm palindrome.
 - iii. In equal temperament, there are 12 individual pitch-classes numbered 0-11.
 - iv. When employed as a drone, the unison can be given linear momentum through the use of dynamic, timbre, duration, register, density and intensity.
 - v. When employed as an ostinato, the added manipulation of rhythm and articulation offers further dimensions for establishing linear motion (goal) and introduces the possibilities for musical expression on a single note.
 - vi. Messiaen employs the unison (octave) to compose homophonic monophony (tutti unison and octave) within his *Quartet for the End of Time*.
 - vii. Therefore, the unison can be used across the ensemble as a vertical or horizontal palindrome, with or without linear motion.
 - viii. The unison ostinato is useful for exploring palindromic (non-retrogradable) rhythms and potential sketches for augmenting and diminishing phrasing.

2. Intervals, Interval-types and Interval-classes – Notes in Pairs (linear or vertical):
 - i. As isolated units, all intervals (paired-pitches, interval-types) are self-contained static equidistant palindromes.
 - ii. There are twelve interval-types (intervals): unison/octave, minor 2nd (semi-tone), Major 2nd (whole-tone), minor 3rd, Major 3rd, Perfect 4th, tritone (diminished 5th), Perfect 5th, minor 6th, Major 6th, minor 7th and Major 7th.
 - iii. The twelve interval-types divide into six interval-classes.
 - iv. All interval-classes invert to the same interval-class (palindromes).
 - v. Interval-classes function like harmonic inversions in that the inversion completes the span of an octave, therefore the whole-tone inverts to a minor 7th and a minor third inverts to a Major 6th etc.
 - vi. Interval classes 1 – 5 are all asymmetrical palindromes (mirror opposites).
 - vii. Interval class 6 is a symmetrical palindrome (mirror symmetry).
 - viii. Interval classes 1, 2, 3, 4 and 6 can all be used in multiple to complete an octave equidistant scale; a self-contained and symmetrical cyclic palindrome:
 - a. Interval Class 1 (semi-tone/Major 7th): 12 semi-tones = 1 octave.
 - b. Interval Class 2 (whole-tone/minor 7th): 6 whole-tones = 1 octave.
 - c. Interval Class 3 (minor 3rd/Major 6th): 4 minor thirds = 1 octave.
 - d. Interval Class 4 (Major 3rd/minor 6th): 3 Major thirds = 1 octave.
 - e. Interval Class 6 (tritone/diminished 5th): 2 tritones = 1 octave.

3. Interval Class Five: 2-5 (05) <000010> Perfect Fourth: Cycle of Fourths

- i. Interval class five (Perfect 4th/5th) is unique in the fact that multiple repetitions do NOT equate to the span of a single octave.
- ii. The cycle of fourths takes a span of five octaves to return to the unison (octave) and complete the equidistant palindrome (cycle).
- iii. The cycle of fourths passes through all twelve pitch-classes and produces all twelve individual interval-types (without replication) in each rotation.
- iv. The cycle of fourths is therefore an equidistant all-interval consonant twelve-tone scale (chromatic consonance).
- v. The cycle of fourths can be used as a linear (melodic) or vertical (harmonic) self-contained equidistant and infinite pitch palindrome.
- vi. Interval-classes 1 and 5 are of particular note, because they both complete the total chromatic (twelve-tone scale) in one cyclic rotation: Class 1 results in an audible cycle of total dissonance and class 5 results in an audible cycle of total consonance.

4. Interval Class Six: 2-6* [6] (06) <000001>³ Tritone: Mode VII Complement

- i. Interval class 6 (tritone) is unique in the fact that it inverts to the same interval-class AND interval-type.
- ii. The tritone (augmented 4th/diminished 5th) divides the octave in two and is therefore the smallest possible unit through which to present the octave as a perfect symmetrical pitch palindrome around a literal pivot-note (axis).
- iii. It is worth noting that the tritone is theoretically an equidistant division of the octave and is most often heard as a consonant interval in music using 'just-intonation'.
- iv. The name of the interval suggests that it is already a self-contained palindrome: tritone = three tones (also two minor 3^{rds}).
- v. Interval class 6 is the complement to Messiaen's Mode VII (Decachord 10-6)
- vi. Interval class 6 MUST be used as the pivot interval in order to produce a symmetrically inverted all-interval twelve-note (SI AITN) chord or row.
- vii. The tritone is often referred to as the Lydian Augmented Fourth.

5. Trichord 3-12* [4] (048) <000300> Augmented Triad: Mode III Complement

- i. The augmented triad is the only triad to complete the octave and produce an equidistant pitch palindrome. It is often referred to as a vagrant chord.
- ii. The augmented triad only contains interval-class 4: 3 Major 3^{rds} = 1 octave.
- iii. Trichord 3-12 (048) is complement to Messiaen Mode III (Nonachord 9-12).
- iv. Trichord 3-12 is the only trichord to be highlighted within these processes.
- v. Trichord 3-12 is subsumed within Hexachord 6-14.

³ * Indicates that the set is a mirror. The integers in square parenthesis indicate limited transposition.

6. Tetrachord 4-9* [6] (0167) <200022> Double Tritone Tetramirror: Messiaen Mode IV Complement
 - i. Tetrachord 4-9 is the complement to Messiaen Mode IV (Octachord 8-9).
 - ii. Tetrachord 4-9 (0167) is a self-contained palindrome (mirror).

7. All-Interval Tetrachords (AIT): 4-z15 (0146/0256) & 4-z29 (0137/0467) <111111>
 - i. The two All-Interval Tetrachords (AIT) get their name due to the fact they contain all twelve interval-types without replication:
 - a. 0146/0256 (4-z15/4-z15B) <111111> All-Interval Tetrachord 1
 - b. 0137/0467 (4-z29/4-z29B) <111111> All-Interval Tetrachord 2
 - ii. This document has defined that the interval is a micro palindrome and that all interval-classes can produce equidistant pitch palindromes. The AIT are therefore the most succinct way to reference the theoretical concept of intervallic pitch palindromes within a single pitch-class set.
 - iii. The all-interval tetrachords (AIT) are self-contained palindromes.

8. Tetrachord 4-25* [6] (0268) <020202>: French Sixth Chord: Mode VI Complement
 - i. Tetrachord 4-25 is the complement to Messiaen Mode VI (Octachord 8-9).
 - ii. Tetrachord 4-25 (0268) is a self-contained palindrome (mirror).

9. Tetrachord 4-28* [3] (0369) <004002> Diminished Seventh Chord – Vagrant Chord Messiaen Mode II Complement: Not subsumed in any set smaller than octachord
 - i. The Diminished 7th Chord is the only tetrachord to complete the octave and produce an equidistant pitch palindrome: 4 minor 3^{rds} = 1 octave.
 - ii. The Diminished 7th Chord only contains two interval-classes (class 3 and 6) and theoretically divides the octave into equidistant halves and quarters.
 - iii. Tetrachord 4-28 (0369): Complement to Messiaen Mode II (Octachord 8-28).

10. Pentachord 5-35* (02479) <032140> Major Pentatonic Scale – Black Key Pentatonic/Quartal Pentamirror: Dorian mode complement & subsumed within 6-32
 - i. Pentachord 5-35 is complement to Dorian minor mode (Heptachord 7-35).
 - ii. Pentachord 5-35 has obvious tonal/modal foundations and is a literal spelling of the Major Pentatonic Scale: 2-2-3-2-3: C-D-E-G-A-C (02479).
 - iii. Pentachord 5-35 (02479) is a self-contained palindrome (mirror).

- iv. Pentachord 5-35 (02479) can be used as a vehicle for a 'rotating' modulation and an alternative to the classical concept of functioning harmony.
- v. Each transposition of the pentachord produces a different modulation:
 - a. C Major Pentatonic modulates through inversion to A^b Major (minor 6th).
 - b. D Major Pentatonic modulates through inversion to F[#] Major (Major 3rd).
 - c. E Major Pentatonic modulates (inverts) to E Major (unison modulation).
 - d. G Major Pentatonic modulates through inversion to D^b Major (tritone).
 - e. A Major Pentatonic inverts through modulation to B Major (Major 2nd).
- vi. Pentachord 5-35 (02479) – Major Pentatonic Scale is the only pentachord to be highlighted within these processes.

11. Hexachord 6-7* [6] (012678) <420243>: Messiaen Mode V

- i. Hexachord 6-7 is Messiaen's Mode V (literal spelling: 1-1-4-1-1-4)
- ii. Hexachord 6-7 (012678) is a self-contained palindrome (all combinatorial)

12. All-Triad Hexachords (ATH) 6-z17 (012478) and 6-z17B (014678) <322332>:

- i. All-Triad Hexachords (ATH) are named for the fact they contain all twelve prime-form trichords (without replication) within the span of an octave:
 - a. 3-1* (012) <210000> BACH/Chromatic Trimirror
 - b. 3-2 (013) <111000> Phrygian Trichord
 - c. 3-3 (014) <101100> Major-minor Trichord
 - d. 3-4 (015) <100110> Incomplete Major Seventh Chord
 - e. 3-5 (016) <100011> Rite Chord/Tritone Fourth
 - f. 3-6* (024) <020100> Whole-tone Trichord
 - g. 3-7 (025) <011010> Incomplete minor seventh Chord
 - h. 3-8 (026) <010101> Incomplete Dominant Seventh Chord – Italian Sixth
 - i. 3-9* (027) <010020> Quartal Trichord
 - j. 3-10* (036) <002001> Diminished Chord
 - k. 3-11 (037) <001110> Major/minor Chord (047)
 - l. 3-12* [4] (048) <000300> Augmented Chord
- ii. The ATH are self-contained palindromes (combinatorial).
- iii. The ATH are subsumed in prime form within the Symmetrically Inverted All-Interval Twelve-Note (SI AITN) Chords: 1, 3, 58 & 60.
- iv. The SI AITN chords were the theoretical starting point for this research. They offered a flexible way to conceive the largest compositional units (12-tone scales) as symmetrical pitch palindromes on vertical and linear axis.
- v. There are many SI AITN to choose from⁴, but only four subsume the ATH.

⁴ Carter lists 88 but does not include inversions. Researchers have identified further alternates [Link Chords]

- vi. The ATH were therefore fundamental to the cohesive evolution of this research, providing a conceptual central pivot (a conceptual palindrome) to pitch organisation and establishing tangible connections between the largest (SI AITN) and smallest (intervals) compositional units (AIT): Palindromes within palindromes, shapes within shapes (Matryoshka Principle):
- vii. SI AITN Chords 1, 3, 58 and 60 all subsume the ATH in prime form.
- viii. The ATH in turn subsume the All-Interval Tetrachords (AIT) in prime form:
 - a. 0137 and 0256 are subsumed within 6-z17 (012478)
 - b. 0146 and 0467 are subsumed within 6-z17B (014678)
- ix. The AIT in turn subsumes all interval-types and classes.
- x. *Rotations and Resonances* for String Quartet explores the ATH as a primary resource for pitch organisation.

13. Hexachord 6-14 (013458/034578) <323430>: Parent chord for SI AITN 1 and 58

- i. Hexachord 6-14 is the parent hexachord for SI AITN 1 and SI AITN 58.
- ii. Hexachord 6-14 (self-contained palindrome) has a twin cell: 6-14B (034578).

14. Hexachord 6-32* (024579) <143250> Arezzo Major Diatonic (literal spelling: 2-2-1-2-2-3), Major Hexamirror, Quartal Hexamirror: Parent hexachord for SI AITN 60

- i. Hexachord 6-32 is the parent hexachord for SI AITN 60.
- ii. With its literal spelling of C-D-E-F-G-A, Hexachord 6-32 has obvious tonal/modal foundations.
- iii. 6-32 can be manipulated to suggest any of the seven modes of Major scale.
- iv. Hexachord 6-32 subsumes Pentachord 5-35 (02479) Major Pentatonic Scale
- v. Hexachord 6-32 (024579) is a self-contained palindrome (combinatorial).

15. Hexachord 6-35* [2] (02468T) <060603> Whole-tone Scale: Messiaen Mode I

- i. Hexachord 6-35 is Messiaen Mode I – Whole-tone scale (2-2-2-2-2-2).
- ii. Hexachord 6-35 is a self-contained palindrome (all-combinatorial).
- iii. *A Quartet of Daydreams* for string quartet explores harmonic and melodic characteristics of whole-tone scale and palindromic pitch-wedges.

16. Dorian minor Mode – Heptachord 7-35* (013568T) <254361>: Modal Palette

- i. Major Diatonic Heptachord/Dominant 13th, Locrian (1-2-2-1-2-2-2), Phrygian (1-2-2-2-1-2-2) and Major Inverse.
- ii. The Dorian Mode is unique as it is the only mode in the tonal system to invert to itself (also produces identical pitches). It is a natural palindrome.
- iii. The complement to 7-35 (Pentachord 5-35) is a Major Pentatonic Scale, which also inverts to produce the same scale-type (alternating modulation).
- iv. Dorian Mode 7-35 (013568T), Hexachord 6-32 (024579), Pentachord 5-35 (02479), Diminished 7th Chord 4-28 (0369) and the Augmented Triad 3-12 (048) are all clearly of tonal/modal foundations.

17. Octachord 8-9* [6] (01236789) <644464>: Messiaen Mode IV

- i. Octachord 8-9 is Messiaen Mode IV (literal spelling: 1-1-1-3-1-1-1-3)
- ii. Octachord 8-9 is a self-contained palindrome.

18. Octachord 8-25* [6] (0124678T) <464644>: Messiaen Mode VI

- i. Octachord 8-25 is Messiaen Mode VI (literal spelling: 1-1-2-2-1-1-2-2)
- ii. Octachord 8-25 is a self-contained palindrome.

19. Octachord 8-28* [3] (0134679T) <448444>: Messiaen Mode II

- i. Octachord 8-28 is Messiaen Mode II (literal spelling: 1-2-1-2-1-2-1-2)
- ii. Octachord 8-28 is also known as the alternating octatonic or diminished scale (auxiliary diminished scale).
- iii. Octachord 8-28 is a self-contained palindrome.

20. Nonachord 9-12* [4] (01245689T) <666963>: Messiaen Mode III

- i. Nonachord 9-12 is Messiaen Mode III (literal spelling: 1-1-2-1-1-2-1-1-2)
- ii. Nonachord 9-12 is a self-contained palindrome.

21. Decachord 10-6* [6] (012346789T) <888885>: Messiaen Mode VII

- i. Decachord 10-6 is Messiaen Mode VII (literal spelling: 1-1-1-1-2-1-1-1-1-2).
- ii. Decachord 10-6 is a self-contained palindrome.

22. Palindromes within Palindromes – Shapes within Shapes – Patterns within Patterns (Matryoshka Principle): Invisible joins ...

- i. SI AITN Chords offer the most flexible way of composing symmetrical twelve-tone palindromes (largest pitch palindromes).
- ii. SI AITN Chords 1, 3, 58 and 60 subsume the All-Triad Hexachord (ATH) in prime form.
- iii. The ATH is the smallest pitch-class set and the most succinct way through which to reference all trichord types highlighted within these processes.
- iv. The ATH subsume the All-Interval Tetrachords (AIT) in prime form.
- v. The AIT is the smallest pitch-class set and the most succinct way through which to reference all interval-types (smallest pitch palindromes).
- vi. The interval (two notes) and the single pitch are obviously the lowest common denominators of this process (micro palindromes).
- vii. All compositional units sit within each other in clear parallels to the Matryoshka Principle, however the recognisable repetition of duplication, imitation, augmentation and diminution of the score contours can be avoided (hidden) quite easily, if and when required.
- viii. The pitch-palette is cohesively composed from micro through to the macro.
- ix. *A Chance Encounter* (Acousmatic) for multiple channel surround sound explores the Matryoshka Principle as a method for flexible 'concrete' form and proportion in acousmatic composition.
- x. *An Acoustic Mandala for the Fourteenth* for Chamber Orchestra and Percussion Ensemble explores symmetrical and asymmetrical patterns within patterns as the primary consideration for pitch organisation and refers to external concepts of geometry (shapes within shapes) for musical form and proportion. The orchestral work also explores the concepts of equidistant scales and chromatic consonance (cycle of fourths etc.).

23. Palindromes can be Symmetrical, Parallel, Duplicates or Asymmetrical:

- i. It is well documented that Leonardo Da Vinci often used mirror-writing in his research, but he also employed mirror reflections (parallels) and mirror opposites (inversions) within his art. Researchers highlight how hand gestures often define a point of axis for the mirror, and through this, Da Vinci employed alternate reflections (angles and planes) and mirror imitations that produced asymmetrical (visual) palindromes.

24. Palindromes are theoretically both infinitely large and infinitely small, they can be Mirror Opposites, Mirror Reflections, Mirror Inversions and Mirror Retrogrades.

25. Palindromes can be panoptic (all axis), linear (melodic) and/or vertical (harmonic).

Palindrome Pitch Matrices

4-z15: (0146/0256) <111111> All-Interval Tetrachord (AIT) 1 and 2 (Inverts to 4-z15)

0	1	4	6
11	0	3	5
8	9	0	2
6	7	10	0

C	C [#]	E	F [#]
B	C	E ^b	F
G [#]	A	C	D
F [#]	G	B ^b	C

4-z29: (0137/0467) <111111> All-Interval Tetrachord (AIT) 3 and 4 (Inverts to 4-z29)

0	1	3	7
11	0	2	6
9	10	0	4
5	6	8	0

C	D ^b	E ^b	G
B	C	D	F [#]
A	B ^b	C	E
F	G ^b	A ^b	C

4-9* [6]: (0167) <200022> Double Tritone Tetramirror: Messiaen Mode IV Complement

0	1	6	7
11	0	5	6
6	7	0	1
5	6	11	0

C	C [#]	F [#]	G
B	C	G	F [#]
F [#]	G	C	D ^b
F	F [#]	B	C

4-25* [6]: (0268) <020202> French Sixth Chord: Mode VI Complement

0	2	6	8
10	0	4	6
6	8	0	2
4	6	10	0

C	D	F [#]	G [#]
A [#]	C	E	F [#]
F [#]	G [#]	C	D
E	F [#]	A [#]	C

4-28* [3]: (0369) <004002> Diminished Seventh Chord: Messiaen Mode II Complement

0	3	6	9
9	0	3	6
6	9	0	3
3	6	9	0

C	D [#]	F [#]	A
A	C	D [#]	F [#]
F [#]	A	C	D [#]
D [#]	F [#]	A	C

Note: 4-28 (0369) not subsumed within any larger set listed (smaller than an octachord).

5-35*: (02479) <032140> Major Pentatonic Scale (2-2-3-2-3): Black Key
 Pentatonic/Quartal Pentamirror: Dorian Mode Complement and subsumed within 6-32

0	2	4	7	9
10	0	2	5	7
8	10	0	3	5
5	7	9	0	2
3	5	7	10	0

C	D	E	G	A
B ^b	C	D	F	G
A ^b	B ^b	C	E ^b	F
F	G	A	C	D
E ^b	F	G	B ^b	C

Note: 5-35 (02479) has tonal/modal origins.

6-z17: (012478/014678) <322332> All-Triad Hexachord (ATH): Inverts to 6-z17
 012478 subsume 0137&0256 [1248=0137, 2478=0256], 014678 subsume 0146&0467

0	1	2	4	7	8
11	0	1	3	6	7
10	11	0	2	5	6
8	9	10	0	3	4
5	6	7	9	0	1
4	5	6	8	11	0

C	C [#]	D	E	G	A ^b
B	C	C [#]	E ^b	F [#]	G
B ^b	B	C	D	F	F [#]
A ^b	A	B ^b	C	E ^b	E
F	F [#]	G	A	C	C [#]
E	F	F [#]	A ^b	B	C

6-14: (013458/034578) <323430> Combinatorial P (6): Inversion & Complement = 6-14

0	1	3	4	5	8
11	0	2	3	4	7
9	10	0	1	2	5
8	9	11	0	1	4
7	8	10	11	0	3
4	5	7	8	9	0

C	C [#]	D [#]	E	F	G [#]
B	C	D	D [#]	E	G
A	B ^b	C	D ^b	D	F
G [#]	A	B	C	C [#]	E
G	A ^b	B ^b	B	C	E ^b
E	F	G	G [#]	A	C

Note: 6-14 (013458) is parent for SI AITN 1 & 58.

6-32*: (024579) <143250> Arezzo major Diatonic (2-2-1-2-2-3), major hexamirror, quartal hexamirror, 1st ord. All-combinatorial P (6), I (3), RI (9): Modal/Tonal Family

0	2	4	5	7	9
10	0	2	3	5	7
8	10	0	1	3	5
7	9	11	0	2	4
5	7	9	10	0	2
3	5	7	8	10	0

C	D	E	F	G	A
B ^b	C	D	E ^b	F	G
A ^b	B ^b	C	D ^b	E ^b	F
G	A	B	C	D	E
F	G	A	B ^b	C	D
E ^b	F	G	A ^b	B ^b	C

Note: 6-32* (024579) is parent for SI AITN 60.

7-35*: (013568T) <254361> Dorian minor Mode:

Only mode to invert to itself (unique in the modal/tonal system)

Spelling: 2-1-2-2-2-1-2 (C-D-E^b-F-G-A-B^b-C)

Heptachord 7-35* (013568T) <254361> Palindrome: Inverts to 7-35

Complement: 5-35 (02479) Major Pentatonic Scale <032140> Palindrome: Inverts to 5-35

0	2	3	5	7	9	10	0
10	0	1	3	5	7	8	10
9	11	0	2	4	6	7	9
7	9	10	0	2	4	5	7
5	7	8	10	0	2	3	5
3	5	6	8	10	0	1	3
2	4	5	7	9	11	0	2
0	2	3	5	7	9	10	0

C	D	E ^b	F	G	A	B ^b	C
B ^b	C	D ^b	E ^b	F	G	A ^b	B ^b
A	B	C	D	E	F [#]	G	A
G	A	B ^b	C	D	E	F	G
F	G	A ^b	B ^b	C	D	E ^b	F
E ^b	F	G ^b	A ^b	B ^b	C	D ^b	E ^b
D	E	F	G	A	B	C	D
C	D	E ^b	F	G	A	B ^b	C

Note: Strong references to the Dorian mode are present within SI AITN 60 (lower hexachord = C Dorian, upper hexachord = B Dorian) and the Hexachord 6-32 (024579). SI AITN 60 also passes through a semi-tone modulation from C Dorian – B Dorian (0 = C).

Note: Dorian Mode (7-35) is related to SI AITN 60 and Hexachord 6-32.

SI AITN 1: 27431 6 E985T – 27431 6 E985T (6-14): Subsumes 6-Z17 twice

Literal Spelling: 029145 ET7386 = C – D – A – C[#] – E – F – B – A[#] – G – D[#] – G[#] – F[#]

Literal Spelling: 029145 ET7386 = C – D – A – C[#] – E – F – B – A[#] – G – D[#] – G[#] – F[#]

0	2	9	1	4	5	11	10	7	3	8	6
10	0	7	11	2	3	9	8	5	1	6	4
3	5	0	4	7	8	2	1	10	6	11	9
11	1	8	0	3	4	10	9	6	2	7	5
8	10	5	9	0	1	7	6	3	11	4	2
7	9	4	8	11	0	6	5	2	10	3	1
1	3	10	2	5	6	0	11	8	4	9	7
2	4	11	3	6	7	1	0	9	5	10	8
5	7	2	6	9	10	4	3	0	8	1	11
9	11	6	10	1	2	8	7	4	0	5	3
4	6	1	5	8	9	3	2	11	7	0	10
6	8	3	7	10	11	5	4	1	9	2	0

Note: SI AITN 1 and 58 share the same parent Hexachord 6-14 (013458).

SI AITN 3: 27491 6 E385T/27491 6 E385T (6-1) Subsumes 6-Z17 twice

Literal Spelling: 0291TE 547386 = C – D – A – C[#] – A[#] – B – F – E – G – D[#] – G[#] – F[#]

Literal Spelling: 0291TE 547386 = C – D – A – C[#] – A[#] – B – F – E – G – D[#] – G[#] – F[#]

0	2	9	1	10	11	5	4	7	3	8	6
10	0	7	11	8	9	3	2	5	1	6	4
3	5	0	4	1	2	8	7	10	6	11	9
11	1	8	0	9	10	4	3	6	2	7	5
2	4	11	3	0	1	7	6	9	5	10	8
1	3	10	2	11	0	6	5	8	4	9	7
7	9	4	8	5	6	0	11	2	10	3	1
8	10	5	9	6	7	1	0	3	11	4	2
5	7	2	6	3	4	10	9	0	8	1	11
9	11	6	10	7	8	2	1	4	0	5	3
4	6	1	5	2	3	9	8	11	7	0	10
6	8	3	7	4	5	11	10	1	9	2	0

Note: The parent hexachord for SI AITN 3 (6-1) is a chromatic tritone, so one can presume this will be the most dissonant and chromatic material.

SI AITN58: 21497 6 538ET – 21497 6 538ET (6-14) Subsumes 6-Z17 twice

Literal Spelling: 02374E 5T1986 = C – D – D[#] – G – E – B – F – B^b – D^b – A – G[#] – F[#]

Literal Spelling: 02374E 5T1986 = C – D – D[#] – G – E – B – F – B^b – D^b – A – G[#] – F[#]

0	2	3	7	4	11	5	10	1	9	8	6
10	0	1	5	2	9	3	8	11	7	6	4
9	11	0	4	1	8	2	7	10	6	5	3
5	7	8	0	9	4	10	3	6	2	1	11
8	10	11	3	0	7	1	6	9	5	4	2
1	3	4	8	5	0	6	11	2	10	9	7
7	9	10	2	11	6	0	5	8	4	3	1
2	4	5	9	6	1	7	0	3	11	10	8
11	1	2	6	3	10	4	9	0	8	7	5
3	5	6	10	7	2	8	1	4	0	11	9
4	6	7	11	8	3	9	2	5	1	0	10
6	8	9	1	10	5	11	4	7	3	2	0

Note: SI AITN 1 and 58 share the same parent Hexachord 6-14 (013458).

SI AITN60: 21437 6 598ET – 21437 6 598ET (6-32) Subsumes 012478 twice

Literal Spelling: 0237T5 E41986 = C – D – E^b – G – B^b – F – B – E – C[#] – A – G[#] – F[#]

Literal Spelling: 0237T5 E41986 = C – D – E^b – G – B^b – F – B – E – C[#] – A – G[#] – F[#]

0	2	3	7	10	5	11	4	1	9	8	6
10	0	1	5	8	3	9	2	11	7	6	4
9	11	0	4	7	2	8	1	10	6	5	3
5	7	8	0	3	10	4	9	6	2	1	11
2	4	5	9	0	7	1	6	3	11	10	8
7	9	10	2	5	0	6	11	8	4	3	1
1	3	4	8	11	6	0	5	2	10	9	7
8	10	11	3	6	1	7	0	9	5	4	2
11	1	2	6	9	4	10	3	0	8	7	5
3	5	6	10	1	8	2	7	4	0	11	9
4	6	7	11	2	9	3	8	5	1	0	10
6	8	9	1	4	11	5	10	7	3	2	0

Note: SI AITN 60 parent Hexachord 6-32 (024579) has modal/tonal origins and is closely related to 5-35 and 7-35.

Modes of Limited Transposition

Olivier Messiaen

Messiaen Mode I (Whole-tone Scale): Two transpositions (two modes)

Literal Spelling: 2-2-2-2-2-2 (C-D-E-F[#]-G[#]-A[#]-C)

Hexachord 6-35* [2] (02468T) <060603> Palindrome: Inverts to 6-35: Complement: 6-35

Messiaen Mode II (Octatonic or Diminished Scale): Three transpositions (three modes)

Literal Spelling: 1-2-1-2-1-2-1-2 (auxiliary diminished) OR 2-1-2-1-2-1-2-1 (diminished)

Octachord 8-28* [3] (0134679T) <448444> Palindrome: Inverts to 8-28

Complement: 4-28* [3] (0369) Diminished 7th Chord <004002> Inverts to 4-28

Note: Subsumes AIT: 0137/0467 - 0146/0256 [1367 = 0256]

Messiaen Mode III: Four transpositions (four modes)

Literal Spelling: 2-1-1-2-1-1-2-1-1 (C-D-E^b-E-F[#]-G-A-B^b-B-C)

Nonachord 9-12* [4] (01245689T) <666963> Palindrome: Inverts to 9-12

Complement: 3-12* [4] (048) Augmented Triad <000300> Palindrome: Inverts to 3-12

Messiaen Mode IV: Six transpositions (six modes)

Literal Spelling: 1-1-1-3-1-1-1-3 (C-D^b-D-E^b-G^b-G-A^b-A-C)

Octachord 8-9* [6] (01236789) <644464> Palindrome: Inverts to 8-9

Complement: Tetrachord 4-9 (0167) <200022> Palindrome: Inverts to 4-9

Messiaen Mode V: Six transpositions (six modes)

Literal Spelling: 1-1-4-1-1-4 (C-D^b-D-F[#]-G-A^b-C)

Hexachord 6-7* [6] (012678) <420243> Palindrome: Inverts to 6-7: Complement: 6-7

Messiaen Mode VI: Six transpositions (six modes)

Literal Spelling: 1-1-2-2-1-1-2-2 (C-D^b-D-E-F[#]-G-A^b-B^b-C)

Octachord 8-25* [6] (0124678T) <464644> Palindrome: Inverts to 8-25

Complement: 4-25* [6] (0268) French Sixth Chord <020202> Palindrome: Inverts to 4-25

Messiaen Mode VII: Six transpositions (six modes)

Literal spelling: 1-1-1-1-2-1-1-1-1-2 (C-D^b-D-E^b-E-F[#]-G-G[#]-A-B^b-C)

Decachord 10-6* [6] (012346789T) <888885> Palindrome: Inverts to 10-6

Complement: Tritone 2-6* [6] (06) <000001> Palindrome: Tritone inverts to tritone

Messiaen Modes Pitch Matrices

6-35*: Messiaen Mode I (Whole-tone Scale): Two transpositions (two modes)

Literal Spelling: 2-2-2-2-2 (C-D-E-F#-G#-A#-C)

Hexachord 6-35* [2] (02468T) <060603> Palindrome: Inverts to 6-35: Complement: 6-35

0	2	4	6	8	10
10	0	2	4	6	8
8	10	0	2	4	6
6	8	10	0	2	4
4	6	8	10	0	2
2	4	6	8	10	0

C	D	E	F#	G#	B ^b
B ^b	C	D	E	F#	G#
A ^b	B ^b	C	D	E	F#
G ^b	A ^b	B ^b	C	D	E
E	G ^b	A ^b	B ^b	C	D
D	E	G ^b	A ^b	B ^b	C

8-28*: Mode II (Octatonic or Diminished Scale): Three transpositions (three modes)

Literal Spelling: 1-2-1-2-1-2-1-2 (auxiliary diminished) OR 2-1-2-1-2-1-2-1 (diminished)

Octachord 8-28* [3] (0134679T) <448444> Palindrome: Inverts to 8-28

Complement: 4-28* [3] (0369) Diminished 7th Chord <004002> Inverts to 4-28

Note: Subsumes AIT: 0137/0467 - 0146/0256 [1367 = 0256]

0	1	3	4	6	7	9	10
11	0	2	3	5	6	8	9
9	10	0	1	3	4	6	7
8	9	11	0	2	3	5	6
6	7	9	10	0	1	3	4
5	6	8	9	11	0	2	3
3	4	6	7	9	10	0	1
2	3	5	6	8	9	11	0

C	C#	D#	E	F#	G	A	B ^b
B	C	D	E ^b	F	F#	G#	A
A	B ^b	C	D ^b	E ^b	E	F#	G
G#	A	B ^b	C	D	E ^b	F	F#
F#	G	A	B ^b	C	D ^b	E ^b	E
F	F#	G#	A	B	C	D	E ^b
E ^b	E	F#	G	A	B ^b	C	D ^b
D	E ^b	F	F#	G#	A	B	C

9-12*: Messiaen Mode III: Four transpositions (four modes)

Literal Spelling: 2-1-1-2-1-1-2-1-1 (C-D-E^b-E-F#-G-A-B^b-B-C)

Nonachord 9-12* [4] (01245689T) <666963> Palindrome: Inverts to 9-12

Complement: 3-12* [4] (048) Augmented Triad <000300> Palindrome: Inverts to 3-12

0	1	2	4	5	6	8	9	10
11	0	1	3	4	5	7	8	9
10	11	0	2	3	4	6	7	8
8	9	10	0	1	2	4	5	6
7	8	9	11	0	1	3	4	5
6	7	8	10	11	0	2	3	4
4	5	6	8	9	10	0	1	2
3	4	5	7	8	9	11	0	1
2	3	4	6	7	8	10	11	0

8-9*: Messiaen Mode IV: Six transpositions (six modes)

Literal Spelling: 1-1-1-3-1-1-1-3 (C-D^b-D-E^b-G^b-G-A^b-A-C)

Octachord 8-9* [6] (01236789) <644464> Palindrome: Inverts to 8-9

Complement: Tetrachord 4-9 (0167) <200022> Palindrome: Inverts to 4-9

0	1	2	3	6	7	8	9
11	0	1	2	5	6	7	8
10	11	0	1	4	5	6	7
9	10	11	0	3	4	5	6
6	7	8	9	0	1	2	3
5	6	7	8	11	0	1	2
4	5	6	7	10	11	0	1
3	4	5	6	9	10	11	0

C	C [#]	D	D [#]	F [#]	G	G [#]	A
B	C	C [#]	D	F	F [#]	G	G [#]
B ^b	B	C	C [#]	E	F	F [#]	G
A	B ^b	B	C	D [#]	E	F	F [#]
F [#]	G	G [#]	A	C	C [#]	D	D [#]
F	F [#]	G	G [#]	B	C	C [#]	D
E	F	F [#]	G	B ^b	B	C	C [#]
D [#]	E	F	F [#]	A	B ^b	B	C

6-7*: Messiaen Mode V: Six transpositions (six modes)

Literal Spelling: 1-1-4-1-1-4 (C-D^b-D-F[#]-G-A^b-C)

Hexachord 6-7* [6] (012678) <420243> Palindrome: Inverts to 6-7: Complement: 6-7

0	1	2	6	7	8
11	0	1	5	6	7
10	11	0	4	5	6
6	7	8	0	1	2
5	6	7	11	0	1
4	5	6	10	11	0

C	C [#]	D	F [#]	G	G [#]
B	C	C [#]	F	F [#]	G
B ^b	B	C	E	F	F [#]
F [#]	G	G [#]	C	C [#]	D
F	F [#]	G	B	C	C [#]
E	F	F [#]	A [#]	B	C

8-25* Messiaen Mode VI: Six transpositions (six modes)

Literal Spelling: 1-1-2-2-1-1-2-2 (C-D^b-D-E-F[#]-G-A^b-B^b-C)

Octachord 8-25* [6] (0124678T) <464644> Palindrome: Inverts to 8-25

Complement: 4-25* [6] (0268) French Sixth Chord <020202> Palindrome: Inverts to 4-25

0	1	2	4	6	7	8	10
11	0	1	3	5	6	7	9
10	11	0	2	4	5	6	8
8	9	10	0	2	3	4	6
6	7	8	10	0	1	2	4
5	6	7	9	11	0	1	3
4	5	6	8	10	11	0	2
2	3	4	6	8	9	10	0

C	C [#]	D	E	F [#]	G	G [#]	B ^b
B	C	C [#]	D [#]	F	F [#]	G	A
B ^b	B	C	D	E	F	F [#]	G [#]
G [#]	A	B ^b	C	D	D [#]	E	F [#]
F [#]	G	G [#]	B ^b	C	C [#]	D	E
F	F [#]	G	A	B	C	C [#]	D [#]
E	F	F [#]	G [#]	B ^b	B	C	D
D	D [#]	E	F [#]	G [#]	A	B ^b	C

10-6*: Messiaen Mode VII: Six transpositions (six modes)

Literal spelling: 1-1-1-1-2-1-1-1-1-2 (C-D^b-D-E^b-E-F[#]-G-G[#]-A-B^b-C)

Decachord 10-6* [6] (012346789T) <888885> Palindrome: Inverts to 10-6

Complement: Tritone 2-6* [6] (06) <000001> Palindrome: Tritone inverts to tritone

0	1	2	3	4	6	7	8	9	10
11	0	1	2	3	5	6	7	8	9
10	11	0	1	2	4	5	6	7	8
9	10	11	0	1	3	4	5	6	7
8	9	10	11	0	2	3	4	5	6
6	7	8	9	10	0	1	2	3	4
5	6	7	8	9	11	0	1	2	3
4	5	6	7	8	10	11	0	1	2
3	4	5	6	7	9	10	11	0	1
2	3	4	5	6	8	9	10	11	0

C	D ^b	D	E ^b	E	F [#]	G	G [#]	A	B ^b
B	C	D ^b	D	E ^b	F	F [#]	G	G [#]	A
B ^b	B	C	D ^b	D	E	F	F [#]	G	G [#]
A	B ^b	B	C	D ^b	E ^b	E	F	F [#]	G
G [#]	A	B ^b	B	C	D	E ^b	E	F	F [#]
F [#]	G	G [#]	A	B	C	D ^b	D	E ^b	E
F	F [#]	G	G [#]	A	B	C	D ^b	D	E ^b
E	F	F [#]	G	G [#]	B ^b	B	C	D ^b	D
E ^b	E	F	F [#]	G	A	B ^b	B	C	D ^b
D	E ^b	E	F	F [#]	G [#]	A	B ^b	B	C

List of Pitch-Class Sets

1. Unison (Octave): The details of the individual note.
2. Intervals, Interval-types and Interval-classes
3. Interval Class Five: 2-5 (05) <000010> Perfect Fourth: Cycle of Fourths
4. Interval Class Six: 2-6* [6] (06) <000001> Tritone: Mode VII Complement
5. Trichord 3-12* [4] (048) <000300> Augmented Triad: Mode III Complement
6. Tetrachord 4-9* [6] (0167) <200022> Double Tritone Tetramirror: Messiaen Mode IV Complement
7. All-Interval Tetrachords (AIT):
 - i. 4-z15/4-z15B (0146/0256) <111111>
 - ii. 4-z29/4-z29B (0137/0467) <111111>
8. Tetrachord 4-25* [6] (0268) <020202> French Sixth Chord: Mode VI Complement
9. Tetrachord 4-28* [3] (0369) <004002> Diminished Seventh Chord – Vagrant Chord
Messiaen Mode II Complement: Not subsumed in any set smaller than octachord
10. Pentachord 5-35* (02479) <032140> Major Pentatonic Scale – Black Key
Pentatonic/Quartal Pentamirror: Dorian mode complement & subsumed within 6-32
11. Hexachord 6-7* [6] (012678) <420243> Messiaen Mode V
12. Hexachord 6-14 (013458/034578) <323430> Parent chord for SI AITN 1 and 58
13. All-Triad Hexachord (ATH):
 - i. 6-z17 (012478) <322332>
 - ii. 6-z17B (014678) <322332>
14. Hexachord 6-32* (024579) <143250> Arezzo Major Diatonic (literal spelling: 2-2-1-2-2-3), Major Hexamirror, Quartal Hexamirror: Parent hexachord for SI AITN 60
15. Hexachord 6-35* [2] (02468T) <060603> Whole-tone Scale: Messiaen Mode I
16. Heptachord 7-35* (013568T) <254361> Dorian minor Mode: Modal Palette
17. Octachord 8-9* [6] (01236789) <644464> Messiaen Mode IV
18. Octachord 8-25* [6] (0124678T) <464644> Messiaen Mode VI
19. Octachord 8-28* [3] (0134679T) <448444> Messiaen Mode II
20. Nonachord 9-12* [4] (01245689T) <666963>: Messiaen Mode III
21. Decachord 10-6* [6] (012346789T) <888885>: Messiaen Mode VII

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