

# **The Musical Palindromes of Olivier Messiaen**

## **Modes of Limited Transposition**

The following document contains a short definition and clarification of Messiaen's Modes of Limited Transposition. This paper emerged from a period of contextual, analytical and practical research in planning and preparation for a large orchestral composition involving musical palindromes.

Pitch palindromes, and the potential for exploring palindromes within palindromes, adds another musical stratum within an evolving concept of Matryoshka Form<sup>1</sup> and self-similar replication which emerged from recent consideration of pitch organisation and proportional subcomponents of internal architecture relative to the study of panoptic non-linear resonances, fractals and chaos theory.

Messiaen's Modes of Limited Transposition are all self-contained pitch palindromes ...

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<sup>1</sup> A design paradigm indicating the existence of object within similar object, originating from the Russian 'nested' dolls design. It is important to note that Matryoshka dolls are traditionally hand painted. They are not exact replications, but self-similar replications. Their subtle differences make each unique ...

## The Musical Palindromes of Olivier Messiaen

Modes of Limited Transposition

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1. Unison (Octave) – The details of the individual note:
  - i. The unison is a static palindrome frozen in time (vertical time).
  - ii. An isolated pitch is therefore (theoretically) the smallest unit through which to present a symmetrical, self-contained, vertical and linear, pitch and rhythm palindrome.
  - iii. Messiaen employs the unison (octave) to compose homophonic monophony (tutti unison and octave) within his *Quartet for the End of Time*.
  - iv. Therefore, the unison can be used across the ensemble as a vertical or horizontal palindrome, with or without linear motion.
  - v. The unison ostinato is useful for exploring palindromic (non-retrogradable) rhythms and potential sketches for augmenting and diminishing phrasing.
  
2. Intervals, Interval-types and Interval-classes – Notes in Pairs (linear or vertical):
  - i. As isolated units, all intervals (paired-pitches, interval-types) are self-contained static equidistant palindromes.
  - ii. All interval-classes invert to the same interval-class (palindromes).
  
3. Interval Class Six: 2-6\* [6] (06) <000001><sup>2</sup> Tritone: Mode VII Complement
  - i. Interval class 6 (tritone) is unique in the fact that it inverts to the same interval-class AND interval-type.
  - ii. The tritone (augmented 4<sup>th</sup>/diminished 5<sup>th</sup>) divides the octave in two and is therefore the smallest possible unit through which to present the octave as a perfect symmetrical pitch palindrome around a literal pivot-note (axis).
  - iii. Although microtonality is not considered within the processes for this piece, it is worth note that the tritone is theoretically an equidistant division of the octave often heard as a consonant interval in music using 'just-intonation'.
  - iv. The name of the interval suggests that it is already a self-contained palindrome: tritone = three tones (also two minor 3<sup>rd</sup>s).
  - v. Interval class 6 is the complement to Messiaen's Mode VII (Decachord 10-6)
  - vi. Interval class 6 MUST be used as the pivot interval in order to produce a symmetrically inverted all-interval twelve-note (SI AITN) chord or row.
  - vii. The tritone is often referred to as the Lydian Augmented Fourth.

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<sup>2</sup> \* Indicates that the set is a mirror. The integers in square parenthesis indicate limited transposition.

4. Trichord 3-12\* [4] (048) <000300> Augmented Triad: Mode III Complement
  - i. The augmented triad is the only triad to complete the octave and produce an equidistant pitch palindrome. It is often referred to as a vagrant chord.
  - ii. The augmented triad only contains interval-class 4: 3 Major 3<sup>rds</sup> = 1 octave.
  - iii. Trichord 3-12 (048) is complement to Messiaen Mode III (Nonachord 9-12).
  - iv. Trichord 3-12 is subsumed within 6-14.
  
5. Tetrachord 4-9\* [6] (0167) <200022> Double Tritone Tetramirror: Messiaen Mode IV Complement
  - i. Tetrachord 4-9 is the complement to Messiaen Mode IV (Octachord 8-9).
  - ii. Tetrachord 4-9 (0167) is a self-contained palindrome (mirror).
  
6. Tetrachord 4-25\* [6] (0268) <020202>: French Sixth Chord: Mode VI Complement
  - i. Tetrachord 4-25 is the complement to Messiaen Mode VI (Octachord 8-9).
  - ii. Tetrachord 4-25 (0268) is a self-contained palindrome (mirror).
  
7. Tetrachord 4-28\* [3] (0369) <004002> Diminished Seventh Chord – Vagrant Chord  
Messiaen Mode II Complement: Not subsumed in any set smaller than octachord
  - i. The Diminished 7<sup>th</sup> Chord is the only tetrachord to complete the octave and produce an equidistant pitch palindrome: 4 minor 3<sup>rds</sup> = 1 octave.
  - ii. The Diminished 7<sup>th</sup> Chord only contains two interval-classes (class 3 and 6) and theoretically divides the octave into equidistant halves and quarters.
  - iii. Tetrachord 4-28 (0369): Complement to Messiaen Mode II (Octachord 8-28).
  
8. Hexachord 6-7\* [6] (012678) <420243>: Messiaen Mode V
  - i. Hexachord 6-7 is Messiaen's Mode V (literal spelling: 1-1-4-1-1-4)
  - ii. Hexachord 6-7 (012678) is a self-contained palindrome (all combinatorial)

9. Hexachord 6-35\* [2] (02468T) <060603> Whole-tone Scale: Messiaen Mode I

- i. Hexachord 6-35 is Messiaen Mode I – Whole-tone scale (2-2-2-2-2-2).
- ii. Hexachord 6-35 is a self-contained palindrome (all-combinatorial).

10. Octachord 8-9\* [6] (01236789) <644464>: Messiaen Mode IV

- i. Octachord 8-9 is Messiaen Mode IV (literal spelling: 1-1-1-3-1-1-1-3)
- ii. Octachord 8-9 is a self-contained palindrome.

11. Octachord 8-25\* [6] (0124678T) <464644>: Messiaen Mode VI

- i. Octachord 8-25 is Messiaen Mode VI (literal spelling: 1-1-2-2-1-1-2-2)
- ii. Octachord 8-25 is a self-contained palindrome.

12. Octachord 8-28\* [3] (0134679T) <448444>: Messiaen Mode II

- i. Octachord 8-28 is Messiaen Mode II (literal spelling: 1-2-1-2-1-2-1-2)
- ii. Octachord 8-28 is also known as the alternating octatonic or diminished scale (auxiliary diminished scale).
- iii. Octachord 8-28 is a self-contained palindrome.

13. Nonachord 9-12\* [4] (01245689T) <666963>: Messiaen Mode III

- i. Nonachord 9-12 is Messiaen Mode III (literal spelling: 1-1-2-1-1-2-1-1-2)
- ii. Nonachord 9-12 is only Nonachord highlighted within these processes.
- iii. Nonachord 9-12 is a self-contained palindrome.

14. Decachord 10-6\* [6] (012346789T) <888885>: Messiaen Mode VII

- i. Decachord 10-6 is Messiaen Mode VII (literal spelling: 1-1-1-1-2-1-1-1-1-2).
- ii. Decachord 10-6 is the only Decachord highlighted within these processes.
- iii. Decachord 10-6 is a self-contained palindrome.

## Modes of Limited Transposition

Olivier Messiaen

Messiaen Mode I (Whole-tone Scale): Two transpositions (two modes)

Literal Spelling: 2-2-2-2-2-2 (C-D-E-F#-G#-A#-C)

Hexachord 6-35\* [2] (02468T) <060603> Palindrome: Inverts to 6-35: Complement: 6-35

Messiaen Mode II (Octatonic or Diminished Scale): Three transpositions (three modes)

Literal Spelling: 1-2-1-2-1-2-1-2 (auxiliary diminished) OR 2-1-2-1-2-1-2-1 (diminished)

Octachord 8-28\* [3] (0134679T) <448444> Palindrome: Inverts to 8-28

Complement: 4-28\* [3] (0369) Diminished 7<sup>th</sup> Chord <004002> Inverts to 4-28

**Note:** Subsumes AIT: 0137/0467 - 0146/0256 [1367 = 0256]

Messiaen Mode III: Four transpositions (four modes)

Literal Spelling: 2-1-1-2-1-1-2-1-1 (C-D-E<sup>b</sup>-E-F#-G-A-B<sup>b</sup>-B-C)

Nonachord 9-12\* [4] (01245689T) <666963> Palindrome: Inverts to 9-12

Complement: 3-12\* [4] (048) Augmented Triad <000300> Palindrome: Inverts to 3-12

Messiaen Mode IV: Six transpositions (six modes)

Literal Spelling: 1-1-1-3-1-1-1-3 (C-D<sup>b</sup>-D-E<sup>b</sup>-G<sup>b</sup>-G-A<sup>b</sup>-A-C)

Octachord 8-9\* [6] (01236789) <644464> Palindrome: Inverts to 8-9

Complement: Tetrachord 4-9 (0167) <200022> Palindrome: Inverts to 4-9

Messiaen Mode V: Six transpositions (six modes)

Literal Spelling: 1-1-4-1-1-4 (C-D<sup>b</sup>-D-F#-G-A<sup>b</sup>-C)

Hexachord 6-7\* [6] (012678) <420243> Palindrome: Inverts to 6-7: Complement: 6-7

Messiaen Mode VI: Six transpositions (six modes)

Literal Spelling: 1-1-2-2-1-1-2-2 (C-D<sup>b</sup>-D-E-F#-G-A<sup>b</sup>-B<sup>b</sup>-C)

Octachord 8-25\* [6] (0124678T) <464644> Palindrome: Inverts to 8-25

Complement: 4-25\* [6] (0268) French Sixth Chord <020202> Palindrome: Inverts to 4-25

Messiaen Mode VII: Six transpositions (six modes)

Literal spelling: 1-1-1-1-2-1-1-1-1-2 (C-D<sup>b</sup>-D-E<sup>b</sup>-E-F#-G-G#-A-B<sup>b</sup>-C)

Decachord 10-6\* [6] (012346789T) <888885> Palindrome: Inverts to 10-6

Complement: Tritone 2-6\* [6] (06) <000001> Palindrome: Tritone inverts to tritone

## Messiaen Modes Pitch Matrices

**6-35\*: Messiaen Mode I (Whole-tone Scale):** Two transpositions (two modes)

Literal Spelling: 2-2-2-2-2-2 (C-D-E-F#-G#-A#-C)

Hexachord 6-35\* [2] (02468T) <060603> Palindrome: Inverts to 6-35: Complement: 6-35

0	2	4	6	8	10
10	0	2	4	6	8
8	10	0	2	4	6
6	8	10	0	2	4
4	6	8	10	0	2
2	4	6	8	10	0

C	D	E	F#	G#	B <sup>b</sup>
B <sup>b</sup>	C	D	E	F#	G#
A <sup>b</sup>	B <sup>b</sup>	C	D	E	F#
G <sup>b</sup>	A <sup>b</sup>	B <sup>b</sup>	C	D	E
E	G <sup>b</sup>	A <sup>b</sup>	B <sup>b</sup>	C	D
D	E	G <sup>b</sup>	A <sup>b</sup>	B <sup>b</sup>	C

**8-28\*: Mode II (Octatonic or Diminished Scale):** Three transpositions (three modes)

Literal Spelling: 1-2-1-2-1-2-1-2 (auxiliary diminished) OR 2-1-2-1-2-1-2-1 (diminished)

Octachord 8-28\* [3] (0134679T) <448444> Palindrome: Inverts to 8-28

Complement: 4-28\* [3] (0369) Diminished 7<sup>th</sup> Chord <004002> Inverts to 4-28

**Note:** Subsumes AIT: 0137/0467 - 0146/0256 [1367 = 0256]

0	1	3	4	6	7	9	10
11	0	2	3	5	6	8	9
9	10	0	1	3	4	6	7
8	9	11	0	2	3	5	6
6	7	9	10	0	1	3	4
5	6	8	9	11	0	2	3
3	4	6	7	9	10	0	1
2	3	5	6	8	9	11	0

C	C#	D#	E	F#	G	A	B <sup>b</sup>
B	C	D	E <sup>b</sup>	F	F#	G#	A
A	B <sup>b</sup>	C	D <sup>b</sup>	E <sup>b</sup>	E	F#	G
G#	A	B <sup>b</sup>	C	D	E <sup>b</sup>	F	F#
F#	G	A	B <sup>b</sup>	C	D <sup>b</sup>	E <sup>b</sup>	E
F	F#	G#	A	B	C	D	E <sup>b</sup>
E <sup>b</sup>	E	F#	G	A	B <sup>b</sup>	C	D <sup>b</sup>
D	E <sup>b</sup>	F	F#	G#	A	B	C

**4-28\* [3]: Messiaen Mode II Complement:** Three transpositions (three modes)

(0369) <004002> Diminished Seventh Chord:

0	3	6	9
9	0	3	6
6	9	0	3
3	6	9	0

C	D#	F#	A
A	C	D#	F#
F#	A	C	D#
D#	F#	A	C

**Note:** 4-28 (0369) not subsumed within any larger set listed (smaller than an octachord).

**9-12\*: Messiaen Mode III:** Four transpositions (four modes)

Literal Spelling: 2-1-1-2-1-1-2-1-1 (C-D-E<sup>b</sup>-E-F<sup>#</sup>-G-A-B<sup>b</sup>-B-C)

Nonachord 9-12\* [4] (01245689T) <666963> Palindrome: Inverts to 9-12

Complement: 3-12\* [4] (048) Augmented Triad <000300> Palindrome: Inverts to 3-12

0	1	2	4	5	6	8	9	10
11	0	1	3	4	5	7	8	9
10	11	0	2	3	4	6	7	8
8	9	10	0	1	2	4	5	6
7	8	9	11	0	1	3	4	5
6	7	8	10	11	0	2	3	4
4	5	6	8	9	10	0	1	2
3	4	5	7	8	9	11	0	1
2	3	4	6	7	8	10	11	0

**3-12\* [4]: Messiaen Mode III Complement:** Four transpositions (four modes)

(048) <000300> Augmented Triad:

0	4	8
8	0	4
4	8	0

C	E	G <sup>#</sup>
G <sup>#</sup>	C	E
E	G <sup>#</sup>	C

**8-9\*: Messiaen Mode IV:** Six transpositions (six modes)

Literal Spelling: 1-1-1-3-1-1-1-3 (C-D<sup>b</sup>-D-E<sup>b</sup>-G<sup>b</sup>-G-A<sup>b</sup>-A-C)

Octachord 8-9\* [6] (01236789) <644464> Palindrome: Inverts to 8-9

Complement: Tetrachord 4-9 (0167) <200022> Palindrome: Inverts to 4-9

0	1	2	3	6	7	8	9
11	0	1	2	5	6	7	8
10	11	0	1	4	5	6	7
9	10	11	0	3	4	5	6
6	7	8	9	0	1	2	3
5	6	7	8	11	0	1	2
4	5	6	7	10	11	0	1
3	4	5	6	9	10	11	0

C	C <sup>#</sup>	D	D <sup>#</sup>	F <sup>#</sup>	G	G <sup>#</sup>	A
B	C	C <sup>#</sup>	D	F	F <sup>#</sup>	G	G <sup>#</sup>
B <sup>b</sup>	B	C	C <sup>#</sup>	E	F	F <sup>#</sup>	G
A	B <sup>b</sup>	B	C	D <sup>#</sup>	E	F	F <sup>#</sup>
F <sup>#</sup>	G	G <sup>#</sup>	A	C	C <sup>#</sup>	D	D <sup>#</sup>
F	F <sup>#</sup>	G	G <sup>#</sup>	B	C	C <sup>#</sup>	D
E	F	F <sup>#</sup>	G	B <sup>b</sup>	B	C	C <sup>#</sup>
D <sup>#</sup>	E	F	F <sup>#</sup>	A	B <sup>b</sup>	B	C

**4-9\* [6]: Messiaen Mode IV Complement:** Six transpositions (six modes)  
(0167) <200022> Double Tritone Tetramirror:

0	1	6	7
11	0	5	6
6	7	0	1
5	6	11	0

C	C#	F#	G
B	C	G	F#
F#	G	C	D <sup>b</sup>
F	F#	B	C

**6-7\*: Messiaen Mode V:** Six transpositions (six modes)

Literal Spelling: 1-1-4-1-1-4 (C-D<sup>b</sup>-D-F#-G-A<sup>b</sup>-C)

Hexachord 6-7\* [6] (012678) <420243> Palindrome: Inverts to 6-7: Complement: 6-7

0	1	2	6	7	8
11	0	1	5	6	7
10	11	0	4	5	6
6	7	8	0	1	2
5	6	7	11	0	1
4	5	6	10	11	0

C	C#	D	F#	G	G#
B	C	C#	F	F#	G
B <sup>b</sup>	B	C	E	F	F#
F#	G	G#	C	C#	D
F	F#	G	B	C	C#
E	F	F#	A#	B	C

**8-25\* Messiaen Mode VI:** Six transpositions (six modes)

Literal Spelling: 1-1-2-2-1-1-2-2 (C-D<sup>b</sup>-D-E-F#-G-A<sup>b</sup>-B<sup>b</sup>-C)

Octachord 8-25\* [6] (0124678T) <464644> Palindrome: Inverts to 8-25

Complement: 4-25\* [6] (0268) French Sixth Chord <020202> Palindrome: Inverts to 4-25

0	1	2	4	6	7	8	10
11	0	1	3	5	6	7	9
10	11	0	2	4	5	6	8
8	9	10	0	2	3	4	6
6	7	8	10	0	1	2	4
5	6	7	9	11	0	1	3
4	5	6	8	10	11	0	2
2	3	4	6	8	9	10	0

C	D <sup>b</sup>	D	E	F#	G	A <sup>b</sup>	B <sup>b</sup>
B	C	D <sup>b</sup>	E <sup>b</sup>	F	F#	G	A
B <sup>b</sup>	B	C	D	E	F	F#	G#
G#	A	B <sup>b</sup>	C	D	D#	E	F#
F#	G	A <sup>b</sup>	B <sup>b</sup>	C	C#	D	E
F	F#	G	A	B	C	C#	D#
E	F	F#	G#	A#	B	C	D
D	D#	E	F#	G#	A	B <sup>b</sup>	C

**Note:** The matrix for Mode VI is not as symmetrical as it should be – there seems to be an anomaly here for potential exploration/investigation ...



**4-25\* [6]: Messiaen Mode VI Complement:** Six transpositions (six modes)  
 (0268) <020202> French Sixth Chord:

0	2	6	8
10	0	4	6
6	8	0	2
4	6	10	0

C	D	F#	G#
A#	C	E	F#
F#	G#	C	D
E	F#	A#	C

**10-6\*: Messiaen Mode VII:** Six transpositions (six modes)

Literal spelling: 1-1-1-1-2-1-1-1-1-2 (C-D<sup>b</sup>-D-E<sup>b</sup>-E-F#-G-G#-A-B<sup>b</sup>-C)

Decachord 10-6\* [6] (012346789T) <888885> Palindrome: Inverts to 10-6

Complement: Tritone 2-6\* [6] (06) <000001> Palindrome: Tritone inverts to tritone

0	1	2	3	4	6	7	8	9	10
11	0	1	2	3	5	6	7	8	9
10	11	0	1	2	4	5	6	7	8
9	10	11	0	1	3	4	5	6	7
8	9	10	11	0	2	3	4	5	6
6	7	8	9	10	0	1	2	3	4
5	6	7	8	9	11	0	1	2	3
4	5	6	7	8	10	11	0	1	2
3	4	5	6	7	9	10	11	0	1
2	3	4	5	6	8	9	10	11	0

C	D <sup>b</sup>	D	E <sup>b</sup>	E	F#	G	G#	A	B <sup>b</sup>
B	C	D <sup>b</sup>	D	E <sup>b</sup>	F	F#	G	G#	A
B <sup>b</sup>	B	C	D <sup>b</sup>	D	E	F	F#	G	G#
A	B <sup>b</sup>	B	C	D <sup>b</sup>	E <sup>b</sup>	E	F	F#	G
G#	A	B <sup>b</sup>	B	C	D	E <sup>b</sup>	E	F	F#
F#	G	G#	A	B	C	D <sup>b</sup>	D	E <sup>b</sup>	E
F	F#	G	G#	A	B	C	D <sup>b</sup>	D	E <sup>b</sup>
E	F	F#	G	G#	B <sup>b</sup>	B	C	D <sup>b</sup>	D
E <sup>b</sup>	E	F	F#	G	A	B <sup>b</sup>	B	C	D <sup>b</sup>
D	E <sup>b</sup>	E	F	F#	G#	A	B <sup>b</sup>	B	C

**Messiaen Modes**  
List of Pitch-Class Sets

1. Unison (Octave): The details of the individual note.
2. Intervals, Interval-types and Interval-classes ...
3. Interval Class Six: 2-6\* [6] (06) <000001> Tritone: Mode VII Complement
4. Trichord 3-12\* [4] (048) <000300> Augmented Triad: Mode III Complement
5. Tetrachord 4-9\* [6] (0167) <200022> Double Tritone Tetramirror: Mode IV Complement
6. Tetrachord 4-25\* [6] (0268) <020202> French Sixth Chord: Mode VI Complement
7. Tetrachord 4-28\* [3] (0369) <004002> Diminished Seventh Chord – Vagrant Chord  
Messiaen Mode II Complement: Not subsumed in any set smaller than octachord
8. Hexachord 6-7\* [6] (012678) <420243> Messiaen Mode V
9. Hexachord 6-35\* [2] (02468T) <060603> Whole-tone Scale: Messiaen Mode I
10. Octachord 8-9\* [6] (01236789) <644464> Messiaen Mode IV
11. Octachord 8-25\* [6] (0124678T) <464644> Messiaen Mode VI
12. Octachord 8-28\* [3] (0134679T) <448444> Messiaen Mode II
13. Nonachord 9-12\* [4] (01245689T) <666963>: Messiaen Mode III
14. Decachord 10-6\* [6] (012346789T) <888885>: Messiaen Mode VII

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